FREE BOUNDARY PROBLEMS FOR PARABOLIC EQUATIONS

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1. One dimensional problems. Denote by $D_i(T)$ $(1 \leqq i \leqq k)$ a 2-dimensional domain bounded by two curves $x = s_{i-1}(t)$, $x = s_i(t)$ where $0 < t < T$, and by the line segments $t = 0$, $b_{i-1} < x < b_i$ and $t = T$, $s_{i-1}(T) < x < s_i(T)$. Here $s_{i-1}(t) < s_i(t)$, $s_0(t) \equiv b_0$, $s_k(t) \equiv b_k$ where b_0, b_k are constants. Let

$$L_m u \equiv a^m(x, t) \frac{\partial^2 u}{\partial x^2} + b^m(x, t) \frac{\partial u}{\partial x} + c^m(x, t) u - \frac{\partial u}{\partial t} \qquad (m = 1, 2)$$

be parabolic operators with smooth coefficients and with $c^m(x, t) \leqq 0$. Suppose, for definiteness, that k is an even number. Consider the following problem: Find such curves $s_1, \cdots, s_{k-1}$ and functions u_1, u_2, that

$$(1.1) \qquad L_1 u_1 = f_1 \quad \text{in } D_1(T) \cup D_3(T) \cup \cdots \cup D_{k-1}(T),$$

$$(1.2) \qquad L_2 u_2 = f_2 \quad \text{in } D_2(T) \cup D_4(T) \cup \cdots \cup D_k(T),$$

$$(1.3) \qquad u_1(x, 0) = h_1(x) \quad \text{if } b_{i-1} < x < b_i, \qquad i = 1, 3, \cdots, k-1,$$

$$(1.4) \qquad u_2(x, 0) = h_2(x) \quad \text{if } b_{i-1} < x < b_i, \qquad i = 2, 4, \cdots, k,$$

$$(1.5) \qquad \text{either } u_1 = g_1 \quad \text{or} \quad \lambda_1 \frac{\partial u_1}{\partial x} + \mu_1 u_1 = g_1 \quad \text{for } x = b_0, \quad 0 < t < T,$$

$$(1.6) \qquad \text{either } u_2 = g_2 \quad \text{or} \quad \lambda_2 \frac{\partial u_2}{\partial x} + \mu_2 u_2 = g_2 \quad \text{for } x = b_k, \quad 0 < t < T,$$

$$(1.7) \qquad u_1 = u_2 = \Phi\left(\frac{\partial u_1}{\partial x}, \frac{\partial u_2}{\partial x}, s_i, \frac{ds_i}{dt}\right) \quad \text{on } x = s_i(t), \qquad 0 < t < T \qquad (1 \leqq i \leqq k-1),$$

$$(1.8) \qquad \Psi\left(u_1, u_2, \frac{\partial u_1}{\partial x}, \frac{\partial u_2}{\partial x}, s_i, \frac{ds_i}{dt}\right) = 0 \quad \text{on } x = s_i(t), \qquad 0 < t < T \qquad (1 \leqq i \leqq k-1);$$

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