

TEN PROBLEMS IN HILBERT SPACE¹

BY P. R. HALMOS

Dedicated to my teacher and friend Joseph Leo Doob with admiration and affection.

TABLE OF CONTENTS

PREFACE	887
1. CONVERGENT. Does the set of cyclic operators have a non-empty interior?...	889
2. WEIGHTED. Is every part of a weighted shift similar to a weighted shift? . . .	892
3. INVARIANT. If an intransitive operator has an inverse, is its inverse also intransitive?	898
4. TRIANGULAR. Is every normal operator the sum of a diagonal operator and a compact one?.....	901
5. DILATED. Is every subnormal Toeplitz operator either analytic or normal?...	906
6. SIMILAR. Is every polynomially bounded operator similar to a contraction?..	910
7. NILPOTENT. Is every quasinilpotent operator the norm limit of nilpotent ones?.....	915
8. REDUCIBLE. Is every operator the norm limit of reducible ones?.....	919
9. REFLEXIVE. Is every complete Boolean algebra reflexive?.....	922
10. TRANSITIVE. Is every non-trivial strongly closed transitive atomic lattice either medial or self-conjugate?.....	926
BIBLIOGRAPHY.....	931

PREFACE

Machines can write theorems and proofs, and read them. The purpose of mathematical exposition for people is to communicate ideas, not theorems and proofs. Experience shows that almost always the best way to communicate a mathematical idea is to talk about concrete examples and unsolved problems. In what follows I try to communicate some of the basic ideas of Hilbert space theory by discussing a few of its problems.

Nobody, except topologists, is interested in problems about Hilbert space; the people who work in Hilbert space are interested in problems about operators. The problems below are about operators. I do not know the solution of any of them. I guess, however, that

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