HOLOMORPHIC FUNCTIONS ON A BANACH SPACE

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1. Unless otherwise stated we shall use the definitions and notation of [4]. E will always be a complex Banach space and $\mathcal{H}(E)$ will denote the set of all complex valued holomorphic functions on all of E. θ will be a holomorphy type [4, p. 34] and $(\mathcal{H}_\theta(E), \mathfrak{J}_{\omega,\theta})$ will denote the topological vector space of holomorphic functions associated with θ as described in [4, pp. 35, 43]. c_0^+ will denote the set of all sequences of positive real numbers which tend to zero at infinity. $\mathcal{K}$ will denote the set of all convex balanced compact subsets of E. Let B_1 be the closed unit ball of some fixed norm which defines E. $\| \ \|_U$ will denote the Minkowski functional of the subset U of E.

An α-holomorphy type θ is a holomorphy type whose definition depends only on the topological vector space structure of E (i.e., it is independent of the actual norm used to define E) and if $U, V \subset E$ and C is a positive real number such that $CU \subset V$ then for each n we have

$$C^n \|P_n\|_{\theta,U} \leqq \|P_n\|_{\theta,V} \qquad \text{for all } P_n \in \mathcal{P}_\theta({}^nE)$$

where $(\mathcal{P}_\theta({}^nE), \| \ \|_\theta)$ is the $(n+1)$st member of the sequence given by θ (see [4, p. 34]).

DEFINITION 1. (a) *Let θ be an α-holomorphy type then $H_\theta(E)$ is the set of all elements of $\mathcal{H}(E)$ for which*

(1) $\hat{d}^n f(0) \in \mathcal{P}_\theta({}^nE)$.

(2) *For each $K \in \mathcal{K}$, $\exists \epsilon > 0$ such that*

$$\sum_{m=0}^{\infty} \left\| \frac{\hat{d}^m f(0)}{m!} \right\|_{\theta, K+\epsilon B_1} < \infty.$$

(b) *A seminorm p on $H_\theta(E)$ is θ-ported by $K \in \mathcal{K}$ if, for each $\epsilon > 0$, $\exists C(\epsilon) > 0$ such that*

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