MODULAR REPRESENTATIONS OF CLASSICAL LIE ALGEBRAS

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Let K be an algebraically closed field of prime characteristic p. By a classical Lie algebra over K we shall understand a Lie algebra $\mathfrak{g}$ obtained from a complex simple Lie algebra $\mathfrak{g}_C$ by the well-known procedure of Chevalley: see [7] [1], for example. In this note we announce some results on the representation theory of $\mathfrak{g}$ over K; proofs will appear elsewhere. All modules considered will be finite-dimensional and restricted, unless otherwise specified.

0. **Preliminaries.** Denote by Σ the root system of $\mathfrak{g}_C$ relative to a Cartan subalgebra, and let $\Pi=\{\alpha_1 \cdots \alpha_l\}$ be a simple system. Fix a Chevalley basis $\{X_\alpha, \alpha\in\Sigma; H_i, 1\leqq i\leqq l\}$ of $\mathfrak{g}_C$; if $\mathfrak{g}_Z$ is the $\boldsymbol{Z}$-span of this basis, then $\mathfrak{g}=\mathfrak{g}_Z\otimes K$. For convenience, we also denote by X_α, H_i the corresponding elements of $\mathfrak{g}$. Write $\mathfrak{h}=\mathfrak{h}_Z\otimes K$ (=span of the H_i in $\mathfrak{g}$). Kostant's theorem [7, §2] describes the $\boldsymbol{Z}$-form $\mathfrak{U}_Z$ of the universal enveloping algebra of $\mathfrak{g}_C$ generated by all $X_\alpha^m/m!$ $(\alpha\in\Sigma, m\geqq 0)$.

If we let V_λ be the irreducible $\mathfrak{g}_C$-module of highest weight λ, and let $v_0\in V_\lambda$ be a maximal vector (a nonzero vector annihilated by all X_α, $\alpha\in\Pi$), then $\mathfrak{U}_Z v_0$ is an "admissible lattice." Tensoring with K yields a (restricted) $\mathfrak{g}$-module $\overline{V}_\lambda$, which is also a module for the simply connected Chevalley group G constructed from $\mathfrak{g}_C$ over K. If v_0 again denotes the maximal vector $v_0\otimes 1$ in $\overline{V}_\lambda$, then v_0 has weight λ.

Let Λ denote the collection of p^l restricted weights λ characterized by the conditions $0\leqq\lambda(H_i)<p$, $1\leqq i\leqq l$. For each $\lambda\in\Lambda$ let M_λ be the irreducible $\mathfrak{g}$-module of highest weight λ; it is known that M_λ is a homomorphic, but not always isomorphic, image of $\overline{V}_\lambda$. The collection $\mathfrak{M}=\{M_\lambda | \lambda\in\Lambda\}$ exhausts the (isomorphism classes of) irreducible $\mathfrak{g}$-modules. Let $\mathfrak{U}$, $\mathfrak{H}$ be the restricted universal enveloping algebras of $\mathfrak{g}$, $\mathfrak{h}$ over K (u-algebras). (Left) $\mathfrak{U}$-modules correspond precisely to restricted (left) $\mathfrak{g}$-modules. Every u-algebra is a Frobenius algebra, and $\mathfrak{U}$ is even symmetric.

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