ON THE AVERAGE ORDER OF SOME ARITHMETICAL FUNCTIONS

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ABSTRACT. We consider a large class of arithmetical functions generated by Dirichlet series satisfying a functional equation with gamma factors. Our objective is to state some Ω results for the average order of these arithmetical functions.

Our objective here is to state some Ω-theorems on the average order of a class of arithmetical functions.

We indicate very briefly the class of arithmetical functions under consideration. For a more complete description, see [4].

Let $\{a(n)\}$ and $\{b(n)\}$ be two sequences of complex numbers, not identically zero. Let $\{\lambda_n\}$ and $\{\mu_n\}$ be two strictly increasing sequences of positive numbers tending to ∞. Put $s=\sigma+it$ with σ and t both real and suppose that

$$\phi(s) = \sum_{n=1}^{\infty} a(n)\lambda_n^{-s} \quad \text{and} \quad \psi(s) = \sum_{n=1}^{\infty} b(n)\mu_n^{-s}$$

each converge in some half-plane. Let σ_a^* denote the abscissa of absolute convergence of ψ. Put

$$\Delta(s) = \prod_{\nu=1}^{N} \Gamma(\alpha_\nu s + \beta_\nu),$$

where $\alpha_\nu > 0$ and β_ν is complex, $\nu = 1, \cdots, N$. Assume that for sone real number r, ϕ and ψ satisfy the functional equation $\Delta(s)\phi(s) = \Delta(r-s)\psi(r-s)$.

We shall consider the Riesz sum

$$A_q(x) = \frac{1}{\Gamma(q+1)} \sum_{\lambda_n \leqq x} a(n)(x-\lambda_n)^q,$$

where $q \geqq 0$. Let $\alpha = \sum_{\nu=1}^{N} \alpha_\nu$ and define

$$Q_q(x) = \frac{1}{2\pi i}\int_{C_q} \frac{\Gamma(s)\phi(s)x^{s+q}}{\Gamma(s+q+1)}\, ds,$$

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