RELATIVISTIC COVARIANCE OF AN INTERACTING QUANTUM FIELD

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I. **Introduction.** Physicists believe that quantum field theory can describe the interactions between elementary particles. The difficulties in constructing model quantum field theories become more tractable in two dimensional space-time. The best understood two dimensional model is the "$\lambda(\phi^4)_2$ quantum field theory" [1]–[4]. We discuss this model which describes a self interacting boson field $\phi(x, t)$.

Let $\mathcal{B}$ be a bounded open subset of R^2. For $(x, t) \in \mathcal{B}$, the field $\phi(x, t)$ is a sesquilinear form defined on a dense domain $\mathfrak{D}$ in a Hilbert space $\mathcal{H}$. Furthermore ϕ is continuous in (x, t) and satisfies the nonlinear partial differential equation

$$\left\{\frac{\partial^2}{\partial t^2} - \frac{\partial^2}{\partial x^2} + m^2\right\}\phi + 4\lambda\phi^3 = 0. \tag{1}$$

The nonlinear term $(\phi^3)(x, t)$ is defined in [3], and (1) holds on $\mathfrak{D}\times\mathfrak{D}$ as an equation for Schwartz distributions.

For real $f \in \mathcal{C}_0^\infty$, the sesquilinear form

$$\phi(f) = \int \phi(x, t) f(x, t)\, dx\, dt \tag{2}$$

uniquely determines a selfadjoint operator $\phi(f)$ [3]. Let $\mathfrak{A}(\mathcal{B})$ denote the von Neumann algebra

$$\mathfrak{A}(\mathcal{B}) = \{e^{i\phi(f)} : f = \bar{f} \in \mathcal{C}_0^\infty, \operatorname{supp} f \subset \mathcal{B}\}''. \tag{3}$$

One can interpret $\mathfrak{A}(\mathcal{B})$ as the bounded observables in the space-time region $\mathcal{B}$. It is convenient to work with the C^*-algebra $\mathfrak{A}$ of quasilocal observables defined as the norm closure of $\bigcup_{\mathcal{B}\subset R^2}\mathfrak{A}(B)$.

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