NONLINEAR EVOLUTION EQUATIONS IN BANACH LATTICES

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1. Nonlinear operators in a Banach lattice. We recall a Banach lattice is a Banach space X over the real numbers R, which is a lattice under the ordering $\leqq$, satisfying for x, y, z in X and $a \geqq 0$ in R,

(1) $x \leqq y$ implies $x+z \leqq y+z$,

(2) $x \leqq y$ implies $ax \leqq ay$, and

(3) $|x| \leqq |y|$ implies $\|x\| \leqq \|y\|$.

Following [12] we write $x^+ = \sup(x, 0)$ and $x^- = \sup(-x, 0)$, giving $x = x^+ - x^-$ and $|x| = x^+ + x^-$. A positive duality map J is a function from X to the dual X^* with

(1) $(Jx, x) = \|x\|^2$,

(2) $\|Jx\| = \|x\|$,

(3) $(Jx, y) \geqq 0$ if $x \geqq 0$ and $y \geqq 0$, and

(4) $(Jx, y) = 0$ if $x \perp y$ (i.e. $\inf(|x|, |y|) = 0$).

This was introduced in [10].

PROPOSITION 1.1. *A Banach lattice has a positive duality map.*

If g is a convex real valued function on X, then the subgradient $dg: X \to$ subsets of X^* is defined by: w is a $dg(x)$ iff for all u in X, $g(u) \geqq g(x) + (w, u-x)$. A selection of a function $F: X \to$ subsets of Y is a function $f: X \to Y$ with $f(x)$ in $F(x)$ for x in X.

PROPOSITION 1.2. *If X is a Banach lattice with positive duality map J then $y \to 2J(y^+)$ is a selection of the subgradient of $y \to \|y^+\|^2$.*

In the following we study existence of properties of solutions $x(t)$, $t \geqq 0$, of the equation of evolution

$$dx/dt\,(t) = -Ax(t), \qquad x(0) = x_0$$

for a given element x_0 of $D(A) \subset X$, where $A: D(A) \to X$ is a nonlinear operator (i.e. a function). In §§1 and 2, the theory is similar to [3], [4], [5], [7], [8], but is in the Banach lattice setting of [10], [11]. Important properties of A are as follows. See [1] for the similar concept of a T-monotone operator.

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