ON THE MINIMUM NORM PROPERTY OF THE FOURIER PROJECTION IN L^1-SPACES AND IN SPACES OF CONTINUOUS FUNCTIONS[1]

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Introduction. 1. Let $C(T)$ be the Banach space of complex continuous periodic functions on the real line, and $L^1(T)$ the Banach space of complex periodic functions on the real line which are absolutely integrable on $[0, 2\pi)$. For simplicity we shall sometimes denote both spaces by $E(T)$. Let then E_n be the space of trigonometric polynomials $\sum_{k=-n}^{+n} c_k e^{ikt}$, and let $F_n: E(T) \to E_n$ be the Fourier projection, defined by

$$(F_n x)(t) = \sum_{k=-n}^{+n} (x)_k e^{ikt}, \quad \text{where } (x)_k = \frac{1}{2\pi} \int_{-\pi}^{+\pi} x(t) e^{-ikt} dt.$$

Then F_n has minimum norm among the projections $E(T) \to E_n$, [10], [1]. Similar results hold when $E(T)$ is replaced by other Banach spaces of functions, [2], [6].

It has been proved recently that F_n is the unique minimum norm projection $C_R(T) \to E_n$, i.e. that $P = F_n$ if P is a projection $C_R(T) \to E_n$ and $\|P\| = \|F_n\|$, [3], [4]. We prove that F_n is the unique minimum norm projection $L^1(T) \to E_n$, and that neither result can be generalized very much.

It is possible to replace T by any compact abelian group G, the set $\{e^{ikt}: -n \leqq k \leqq +n\}$ of characters of T by any finite set $\{e_\gamma: \gamma \in N \subseteq \hat{G}\}$ of characters of G, and furthermore to consider the mapping $E(G) \to E_N$ given by $x \to x * k$, where $E(G) = C(G)$ or $L^1(G)$, $E_N =$ the linear hull of $\{e_\gamma: \gamma \in N\}$, and $k = \sum_{\gamma \in N} c_\gamma e_\gamma$, $0 \neq c_\gamma \in \mathbf{C}$. It is this generalization we have studied ([7], [8] and [9]); however,

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