SMOOTH MAPS TRANSVERSE TO A FOLIATION

BY ANTHONY PHILLIPS

Communicated by Raoul Bott, November 14, 1969

1. **Introduction.** This article presents a Smale-Hirsch-type classification theorem for smooth maps transverse to a foliation. Let M, W be smooth manifolds, with tangent bundles TM, TW, and let $\mathrm{Hom}(M, W)$, $\mathrm{Hom}(TM, TW)$ represent the spaces of smooth maps $M \to W$ and of fibrewise linear maps $TM \to TW$, where we give to $\mathrm{Hom}(TM, TW)$ the compact-open topology, and to $\mathrm{Hom}(M, W)$ the C^1-compact-open topology; thus the map $d:\mathrm{Hom}(M, W) \to \mathrm{Hom}(TM, TW)$, which associates to each smooth map its differential, is continuous.

Suppose W carries a foliation $\mathfrak{F}$, and let $T\mathfrak{F}$ denote the subbundle of TW tangent to $\mathfrak{F}$ (i.e. the embedding $T\mathfrak{F} \to TW$ is an integrable distribution). Let $\mathrm{Trans}(TM, T\mathfrak{F})$ be the subspace of $\mathrm{Hom}(TM, TW)$ consisting of those maps fibrewise transverse to $T\mathfrak{F}$, and let

$$\mathrm{Trans}(M, \mathfrak{F}) = d^{-1}\,\mathrm{Trans}(TM, T\mathfrak{F}) \subset \mathrm{Hom}\,(M, W).$$

THEOREM 1. *If M is open, then the differential map $d:\mathrm{Trans}(M, \mathfrak{F}) \to \mathrm{Trans}(TM, T\mathfrak{F})$ is a weak homotopy equivalence.*

Suppose now W has a Riemannian metric, so we can define $N\mathfrak{F}$, the normal bundle to $\mathfrak{F}$, to be the bundle whose fibre at $x \in W$ is the orthogonal complement to $T\mathfrak{F}_x$. Then the space $\mathrm{Epi}(TM, N\mathfrak{F})$ of fibrewise linear and surjective maps $TM \to N\mathfrak{F}$ is a subspace and, in fact, a deformation retract, of $\mathrm{Trans}(TM, T\mathfrak{F})$. If we let $p:\mathrm{Hom}(TM, TW) \to \mathrm{Hom}(TM, TW)$ be composition with fibrewise orthogonal projection of TW onto the sub-bundle $N\mathfrak{F}$ then Theorem 1 has the immediate corollary:

THEOREM 2. *If M is open, then the map $p \circ d:\mathrm{Trans}(M, \mathfrak{F}) \to \mathrm{Epi}(TM, N\mathfrak{F})$ is a weak homotopy equivalence.*

REMARKS. Theorem 1, which was proposed to the author by J. W. Milnor, has a special case (where $\mathfrak{F}$ = the foliation by points) the

AMS Subject Classifications. Primary 5736, 5734.

Key Words and Phrases. Foliation, transverse, subbundle, differential map, weak homotopy equivalence, Smale-Hirsch-type classification, covering homotopy property, good position.