PRODUCT FORMULAS FOR $L_n(\pi)$

BY JULIUS L. SHANESON

Communicated by William Browder, December 12, 1969

Introduction. In this note we prove some product formulas for non-simply-connected even dimensional surgery obstructions. This complements [8] (and in fact uses [8] as well as [5]). We also give a simple example of the type of geometric construction that product formulas make possible.

1. **Product formulas.** Let Ω_m be the oriented cobordism classes of oriented, closed, smooth or piecewise-linear (P.L.) manifolds of dimension m. Let π be a finitely presented group, let $w:\pi\to Z_2$ be a homomorphism, and let $L_n^h(\pi, w)$ be the Wall surgery obstruction group for the homotopy problem in dimension $n\geqq 5$ (see [6] or [7]). That is, if $(X^n, \partial X)$ is a manifold, if ξ is a vector bundle over X, if $f:(M, \partial M)\to(X, \partial X)$ is a map of degree one whose restriction induces a homotopy equivalence of boundaries, and if F is a stable framing of $\tau(M)\oplus f^*\xi$; then if $(\pi_1 X, w^1 X)=(\pi, w)$, there is an obstruction $\theta(M, f, F)$ in $L_n^h(\pi, w)$ that vanishes if and only if (M, f, F) is cobordant relative the boundary to (N, g, G), g a homotopy equivalence. The Wall groups satisfy $L_n^h(\pi, w)=L_{n+4}^h(\pi, w)$, and surgery obstructions are invariant under products with complex projective space CP^2. For $n\geqq 6$, every element can be realized as $\theta(M, f, F)$ for a suitable given X and ξ; e.g. $X=K\times I$ and $\xi=\nu(X)$, the normal bundle of X. For low dimensions, obstructions are defined by crossing with CP^2; their vanishing is a necessary condition for the surgery problem to be solvable.

There is a pairing

$$\Omega_m \times L_n^h(\pi, w) \to L_{n+m}^h(\pi, w)$$

defined as follows: Let $\alpha\in\Omega_m$ and let $z\in L_n^h(\pi, w)$. Assume $n\geqq 6$. Choose a simply-connected manifold P representing α, and let X, ξ, M, f, and F be as above so that $\theta(M, f, F)=z$. Let G be the natural framing of $\tau(P)\oplus\nu(P)$, $\nu(P)$ a high dimensional normal bundle of P. Then we make the definition

$$\alpha \times z = \theta(P\times M, 1\times f, G\times F).$$

AMS Subject Classifications. Primary 5705, 5710; Secondary 5747.

Key Words and Phrases. Surgery, surgery obstruction, index, Whitehead torsion, nonlinear representation.