IMBEDDINGS, IMMERSIONS, AND COBORDISM OF DIFFERENTIABLE MANIFOLDS

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1. **Introduction.** The problem of imbedding a closed differentiable manifold M^n in a euclidean space can be weakened through the notion of (modulo 2) cobordism as follows. Is M^n cobordant to a submanifold of R^{n+k}? In this context we can prove an analogue, with improved dimensions, of H. Whitney's theorems [11], [12]. Let $\alpha(n)$ denote the number of ones in the binary expansion of n, and let $n>1$.

THEOREM A. *Any M^n is cobordant to a manifold N^n that imbeds in $R^{2n-\alpha(n)+1}$ and immerses in $R^{2n-\alpha(n)}$.*

For $n\neq 3$ this result is best possible as the examples below show. In some cases we can say more if certain Stiefel-Whitney numbers of M^n are zero. Allow the empty set as a representative of the zero cobordism class. (Thus Theorem A holds for all n.)

THEOREM B. (i) *If n is even ($n\neq 6$) and if $\bar{w}_{\alpha(n)}\cdot\bar{w}_{n-\alpha(n)}(M^n)=0$ then M^n is cobordant to a manifold N^n that imbeds in $R^{2n-\alpha(n)}$ and immerses in $R^{2n-\alpha(n)-1}$.*

(ii) *If $n=2^k$ or 2^k+1 and if $\bar{w}_i\cdot\bar{w}_{n-i}(M^n)=0$ for $0\leqq i\leqq s\leqq 3$ then M^n is cobordant to a manifold N^n that imbeds in R^{2n-s} and immerses in R^{2n-s-1}.*

Let $\mathfrak{N}_*$ denote the modulo 2 cobordism ring, and let $MO(k)$ denote the Thom complex for $O(k)$. There are homomorphisms

$$\Phi(n, k):\pi_{n+k}(MO(k)) \to \mathfrak{N}_n \quad\text{and}\quad \Psi(n, k, N):\pi_{n+k+N}(S^N MO(k)) \to \mathfrak{N}_n.$$

The image of $\Phi(n, k)$ is the set of cobordism classes that can be represented by submanifolds of R^{n+k} and hence coker $\Phi(n, k)=0$ if $k>n-\alpha(n)$ by Theorem A. The image of $\Psi(n, k, N)$ ($N\gg k$) is the set of cobordism classes that can be represented by manifolds which immerse in R^{n+k} (see R. Wells [10]) and hence coker $\Psi(n, k, N)=0$ if $k\geqq n-\alpha(n)$, $N\gg k$.

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