ON BOUNDARY CONDITIONS FOR SYMMETRIC SUBMARKOVIAN RESOLVENTS

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1. **Introduction.** In a recent paper [4], M. Fukushima has established a one-to-one correspondence between symmetric markovian semigroups which satisfy the heat equation on a bounded domain D in Euclidean n-space and certain Dirichlet spaces on the Martin boundary of that domain. In this note we give an extension of his result to a much more general context.

Fukushima considers semigroups with resolvent kernels of the form

$$G_\alpha(x, y) = G_\alpha^0(x, y) + R_\alpha(x, y)$$

where G_α^0 is the "absorbing barrier" or minimal resolvent for Brownian motion on D and $R_\alpha(x, y)$, defined for x and y in D, is a nonnegative, symmetric "α-harmonic" term, i.e. R_α satisfies the equation $\alpha R_\alpha - (1/2)\Delta R_\alpha = 0$ in D as a function of x for fixed y. Also, it is assumed that $\alpha G_\alpha 1 = 1$ in D. We start with a given nonnegative symmetric resolvent G_α^0 which is submarkovian, i.e. $\alpha G_\alpha^0 1 \leqq 1$, and then consider resolvents $G_\alpha \geqq G_\alpha^0$ which are symmetric and submarkovian. The Laplacian operator which plays a central role in Fukushima's work is here replaced by a much more general type of operator A which may not even be a local operator. The main results will be found in Theorems 1–3. Our method of proof is different from that of Fukushima. The details will be published elsewhere.

2. **Preliminaries.** Let (X, dx) be a sigma finite measure space and let $(\ ,\ )_X$ or $(\ ,\ )_{dx,X}$ denote the standard inner product on $L^2(X)$, the Hilbert space of real-valued square integrable functions on X.

2.1. Definition. A symmetric submarkovian resolvent on $L^2(X)$ is a family $\{G_\alpha, \alpha > 0\}$ of bounded linear operators on $L^2(X)$ such that

2.1.1. $G_\alpha f \geqq 0$ a.e. whenever $f \geqq 0$ a.e. and $\alpha G_\alpha 1 \leqq 1$ a.e.

2.1.2. $G_\alpha - G_\beta = (\beta - \alpha) G_\alpha G_\beta$.

2.2. Definition. The measurable function g is a normalized contraction of the measurable function f if $|g(x)| \leqq |f(x)|$ and $|g(x) - g(y)| \leqq |f(x) - f(y)|$ for all x, y in X.

2.3. Definition. A Dirichlet space relative to $L^2(X)$ is a pair $(F, \mathcal{E})$ where

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