A SELF-UNIVERSAL CRUMPLED CUBE WHICH IS NOT UNIVERSAL

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C. E. Burgess and J. W. Cannon [2, §10] have asked whether each self-universal crumpled cube is universal. In this note we give a negative answer to their question by showing that the familiar solid Alexander horned sphere K is not universal. Casler has shown that K is self-universal [3].

A *crumpled cube* C is a space homeomorphic to the union of a 2-sphere S topologically embedded in the 3-sphere S^3 and one of its complementary domains. The *boundary of* C, denoted Bd C, is the image of S under the homeomorphism. A *sewing* h *of two crumpled cubes* C *and* C^* is a homeomorphism of Bd C to Bd C^*. The *space* $C\cup_h C^*$ *given by a sewing* h is the identification space obtained from the (disjoint) union of C and C^* by identifying each point p in Bd C with $h(p)$ in Bd C^*.

A crumpled cube C is *universal* if, for each crumpled cube C^* and each sewing h of C and C^*, the space $C\cup_h C^*$ is topologically equivalent to S^3. Similarly, a crumpled cube C is *self-universal* if $C\cup_f C = S^3$ for each sewing f of C to itself.

1. A bad sewing. In order to define the desired sewing of the solid Alexander horned sphere K to another crumpled cube K^*, we describe an upper semicontinuous decomposition of S^3 into points and almost tame arcs.

Let H_1 and H_2 denote the upper and lower half spaces of E^3, and P the xy-plane. Let A_0 denote a solid double torus embedded in E^3 as shown in Figure 1 such that A_0 intersects P in two disks D_1 and D_2. Letting T_1 and T_2 denote solid double tori embedded in A_0 as shown in Figure 1, we define A_1 as $T_1\cup T_2$. Assuming sets $A_0, A_1, \cdots, A_{n-1}$ have been defined, we let A_n be the union of 2^n solid double tori contained in A_{n-1} such that each double torus T of A_{n-1} contains exactly two components of A_n, which are embedded in T just as T_1 and T_2 are embedded in A_0.

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