GENERALIZED THOM SPECTRA AND TRANSVERSALITY FOR SPHERICAL FIBRATIONS[1]

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1. A Poincaré duality space (abbreviated P.D. space) of dimension $n \geqq 0$ is a finite complex M with the following property.

Let M be embedded in S^{n+k}, k large, and let R be a regular neighborhood; then the inclusion $\partial R \subseteq R$, when converted to a fibration, has fiber a $(k-1)$-sphere.

Similarly a Poincaré cobordism $(W; M_0, M_1)$ of dimension $n+1$ is a triad with the following property.

Let $W; M_0, M_1$ be embedded in $S^{n+k} \times (I; \{0\}, \{1\})$ with relative regular neighborhood R (i.e. $R \cap S^{n+k} \times \{i\} = Q_i$ is a regular neighborhood of M_i in $S^{n+k} \times \{i\}, i = 0, 1$). Let $\bar{\partial} R =$ closure $\partial R - S^{n+k} \times \{0, 1\}$. Then $\bar{\partial} R \subseteq R$ is a $(k-1)$-spherical fibration and $\bar{\partial} R \cap Q_i = \partial Q_i \subseteq Q_i$ is the induced $(k-1)$-spherical fibration.

A P.D. pair $M, \partial M$ is a P.D. cobordism $M; \partial M, \emptyset$. If $W; M_0, M_1$ is a P.D. cobordism then M_0, M_1 are P.D. spaces of one lower dimension. For a P.D. space M let $\nu_k(M)$ be the fibration corresponding to $\partial R \subseteq R$; for a P.D. cobordism $W; M_0, M_1$ let $\nu_k(W; M_0, M_1)$ be the fibration corresponding to $\bar{\partial} R \subseteq R$.

A Generalized Thom Spectrum is a spectrum defined as follows: let $\xi_k : E_k \to B_k$ be a sequence of $(k-1)$-spherical fibrations, $k \geqq 1$. Let $\psi_k : B_k \to B_{k+1}$ be maps covered by spherical-fibration maps $\phi_k : \xi_k \oplus \epsilon \to \xi_{k+1}$.

Let the Thom complex $T(\xi^{xi})$ be the space $\mathfrak{M}_{\xi_k} \cup cE_k$, i.e. the mapping cylinder of $\xi_k : E_k \to B_k$ union the cone on E_k with the top of the mapping cylinder identified with the base of the cone. There are natural maps $\sum T(\xi_k) \to T(\xi_{k+1})$. This forms the generalized Thom spectrum T.

Let S be the spectrum got by taking $B_k = B_{k+1} \cdots =$ point; thus S is the sphere spectrum. If T is any spectrum as above, we assume that there are base points in each B_k, preserved by ψ. This gives an inclusion of spectra $S \subseteq T$.

A T-P.D. space (or simply T-space) is a P.D. space M together with maps of spherical fibrations $f_k : \nu_k(M) \to \xi_k$ so that

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