RIGIDITY OF HYPERSURFACES OF CONSTANT SCALAR CURVATURE

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Communicated by P. E. Thomas, October 30, 1969

Let M_n and $\tilde{M}_{n+1}$ be Riemannian manifolds of dimension n and $n+1$ respectively.

Assume M_n isometrically immersed in $\tilde{M}_{n+1}$.

If each point p of M_n is contained in an open neighborhood $U \subset M_n$ (which may depend on p) such that no open submanifold of U is rigid in $\tilde{M}_{n+1}$, then M_n is called *locally deformable* in $\tilde{M}_{n+1}$.

This concept allowed us to show that the result of Nagano-Takahashi [3] holds without any restriction, i.e. that any homogeneous hypersurface of the Euclidean space E_{n+1} is isometric to the Riemannian product of a p-dimensional sphere and an $n-p$ dimensional Euclidean space.

This result is a consequence of the following

THEOREM 1. *Let M_n be a complete Riemannian manifold of dimension $n \geqq 3$, with constant scalar curvature $K \neq 0$.*

If M_n is locally deformable in the Euclidean space E_{n+1}, then it is isometric to the Riemannian product of a 2-sphere of radius $1/K$ and an $n-2$ dimensional Euclidean space.

The next result gives rigidity of certain hypersurfaces of non-Euclidean space forms.

THEOREM 2. *Let M_n be an n-dimensional Riemannian manifold $n \geqq 4$, with constant scalar curvature K. Assume M_n isometrically immersed in the space form $\tilde{M}_{n+1}(c)$, with $c \neq 0$ and $K \neq n(n-1)c$. Then M_n is rigid in $\tilde{M}_{n+1}(c)$.*

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AMS Subject Classifications. Primary 5374; Secondary 5370.

Key Words and Phrases. Hypersurfaces, homogeneous locally deformable, scalar curvature, Riemannian product, space form, sphere, Euclidean space.

[1] This work has been supported by CNP_q (Rio de Janeiro, Brazil) and EPUSP (São Paulo, Brazil).