BOOK REVIEWS

The algebraic theory of semigroups. Vol. I and II by A. H. Clifford and G. B. Preston. Mathematical Surveys, number 7, American Mathematical Society, Providence, Rhode Island, 1961 and 1967. xvi+244 pp. and xv+350 pp. $11.10 and $14.20.

Semigroups by E. S. Ljapin. Fizmatigiz, Moscow, 1960. vii+447 pp; English translation, Translations of Mathematical Monographs, volume 3, American Mathematical Society, Providence, Rhode Island, 1963. x+487 pp. $22.20.

The theory of finitely generated commutative semigroups by László Rédei. Translated, edited by N. Reilly, Pergamon Press, New York, 1965, xiii+353 pp. $13.50.

Elements of compact semigroups by Karl Heinrich Hofmann and Paul S. Mostert. Charles E. Merrill Books, Inc., Columbus, Ohio, 1966, xiii+384 pp. $15.00.

A semigroup is a set with an associative multiplication. No more and no less.

Recent books on semigroups seem either to attempt to present a survey of the existing theory, such as it is, or to place in one package the author's new research contributions.

Clifford and Preston's two volumes succeed admirably in presenting the algebraic (nontopological and nonordered) theory of semigroups developed up to around 1964. Chapter 1 contains some elementary definitions and theorems, e.g., the structure of cyclic semigroups, that maximal subgroups are disjoint and exist if there are idempotents, and the useful, elementary Ore-Dubreil condition for the embedding of noncommutative semigroups in groups with the elegant proof by Rees.

The next two chapters form the heart of the work. Following Green's 1951 Annals paper, one thinks of the semigroup as the multiplicative part of a ring, and calls two elements L (resp. R, resp. J) equivalent iff they generate the same principal left (resp. right, resp. 2-sided) ideal. These Green equivalence relations are defined for all semigroups. They also define $H=L\cap R$ and $D=L\circ R=R\circ L$. In Chapter 2 Clifford and Preston introduce the Green correspondence between D equivalent H-classes and also show that every maximal subgroup is an H-class and conversely each H-class containing an idempotent is a maximal subgroup. Also an analysis of the D-classes containing an idempotent (so-called regular D-classes) is carried out and two D equivalent maximal subgroups are shown to be isomorphic.