REMARKS CONCERNING Ext* $(M, -)$

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Communicated by Saunders MacLane, September 30, 1969

Let X be a topological space and let $\mathcal{S}$ (respectively $\mathcal{A}$) be the category of sets (respectively abelian groups). Let $\mathcal{S}'$ (respectively $\mathcal{A}'$) be the category of sheaves of sets (respectively abelian groups) based on X, and fix a sheaf M in $\mathcal{A}'$. The graded functor $\mathrm{Ext}^*(M, -)$ $:\mathcal{A}'\to\mathcal{A}$ is computed as the right derived functors of $\mathrm{Hom}(M, -)$, and of course $\mathrm{Ext}^i(M, N)$ classifies i-fold extensions of M by N [6].

One would also like to be able to classify extensions in nonabelian categories of sheaves. Partial success in this direction has been achieved by Gray [5], but he needs to assume restrictions on X as well as on M. In [10], the author applied triple-theoretic [1] techniques to the category of sheaves of R-algebras (R a sheaf of rings), and successfully classified cohomologically singular extensions of an R-algebra P by one of its modules N.

Specifically, if G is the polynomial algebra cotriple lifted to the category of sheaves of R-algebras, if T is the Godement triple = standard construction [3], and if $\mathrm{Der}_R(P, N)$ is the abelian group of global R-derivations from P to N, then the equivalence classes of singular extensions of P by N are in one-one correspondence with the elements of the first homology group of the double complex $\mathrm{Der}_R(G^*P, T^*N)$. In §II of this note we prove that if G is the free abelian group cotriple lifted to $\mathcal{A}'$ then the nth homology group of the double complex $\mathrm{Hom}(G^*M, T^*N)$ is naturally isomorphic to $\mathrm{Ext}^n(M, N)$. The combination of this theorem and the results in [10] indicates a unified approach to the cohomological classification of extensions in many (algebraic) categories of sheaves.

In §I one can find a theorem which is part of the folklore of triple-theoretic cohomology theory, but for which no straightforward proof appears in print. The theorem is: if an abelian category has an injective cogenerator and E is the model-induced triple then $\mathrm{Ext}^*(M, N)$ and the homology of the complex $\mathrm{Hom}(M, E^*N)$ are naturally isomorphic (note that E is not the triple used by Schafer in [8]).

AMS Subject Classifications. Primary 1820; Secondary 1455, 5532.

Key Words and Phrases. Sheaves of sets, sheaves of abelian groups, extensions, triple, cotriple, model-induced triple, homology, double complex.

[1] Partially supported by NSF Grant GP 7506.