ON LOCAL SOLVABILITY OF LINEAR PARTIAL DIFFERENTIAL EQUATIONS

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The title indicates more or less what the talk is going to be about. It is going to be about the problem which is probably the most primitive in partial differential equations theory, namely to know whether an equation does, or does not, have a solution. Even this is meant in the most primitive terms. I would like to begin by explaining what the terms are.

As you all know, the really difficult analysis these days, and perhaps always, is the *global analysis*. Well, the problem that I am going to discuss is purely *local*—in the strictest possible sense: we would like to find out if a linear partial differential equation, with coefficients as smooth as you wish, admits locally a solution. Obviously, in this connection, *negative* results are very important: and negative results about local solvability have global implications. But of course positive results have also their importance. Let us state precisely what is the problem. The partial differential equation under study will be

$$Pu = f \tag{1}$$

and we would like to know whether for given f, defined in the neighborhood of some point, there are solutions u. This is really too vague, so that the first thing we shall do is to make it a little more precise. Let us say that the *differential operator* P is defined in an open set Ω of the Euclidean space R^n and that we wish to solve the equation in Ω; suppose that the right-hand side f is very regular, say $f \in \mathcal{C}^\infty(\Omega)$. Then you would like to know whether there are solutions (defined in Ω). Now, the experience we have acquired since 1950 in the field of linear partial differential equations tells us that if you pose the problem in this way, you will encounter very serious difficulties of *global* nature. After all, Ω is a manifold, in general it possesses a boundary, or points at infinity, and the fact is that the behaviour

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