

QUASI-ISOMETRIC MEASURES AND THEIR APPLICATIONS¹

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1. Introduction

This paper is devoted to certain uses of integration theory which emerge when the measures involved are vector- or operator-valued. These uses, as yet generally unfamiliar, are significant in three ways:

- (1) They yield explicit formulations for many of the representation theorems of functional analysis;

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