SUBGROUPS OF AMALGAMATED FREE PRODUCTS

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Communicated by Michio Suzuki, September 22, 1969

In 1934 Kuroš [5] proved that "a subgroup of a free product of groups is again a free product." Several attempts have been made to extend this to a result about a free product of groups with an amalgamated subgroup, notably [4] and [6]. Theorem 1 here gives to any subgroup of a free product with amalgamated subgroup an induced structure of the same type. We here indicate very briefly the method of proof. Details and related results will appear elsewhere.

DEFINITION 0. Let G_μ be groups, for μ in an index set M, and let G be a group which is isomorphic to a subgroup of each G_μ under given maps δ_μ: $G \to G_\mu$. The *free product of the groups* G_μ *with the amalgamated subgroup* G, denoted $\overline{G} = (*_\mu G_\mu)_G$, is the factor group of the free product $(*_\mu G_\mu)$ with respect to the normal subgroup generated by all elements of the form $\delta_\mu(g)\delta_\nu(g)^{-1}$, where g runs through G and the pair (μ, ν) runs through $M \times M$. That is, $\overline{G}$ is the free product of the G_μ with the subgroups isomorphic to G identified.

THEOREM 1. *Suppose:*

$\overline{G} = (*_\mu G_\mu)_G$ *is a free product of groups with amalgamated subgroup* G, μ *in an index set* M;

$\overline{K} = (*_\mu K_\mu)$ *is a free product of groups,* $\mu \in M$ *the same index set*;

$f: \overline{G} \to \overline{K}$ *is a group homomorphism with* $f(G_\mu) \subset K_\mu$ *for each* μ; *and*

$\overline{H}$ *is a subgroup of* $\overline{G}$ *such that* $f(\overline{H}) = \overline{K}$.

Then:

$\overline{H}$ *is expressible as* $(*_\mu H_\mu)_H$ *with* $f(H_\mu) \subset K_\mu$;

H *is generated as a subgroup of* G *by certain subgroups*

$$g_{0\nu} G_{0\nu} g_{0\nu}^{-1}, \quad G_{0\nu} \subset G_0, \quad g_{0\nu} \in \ker f \subset \overline{G}, \textit{ for } \nu \textit{ in an index set } N_0;$$

Each H_μ *is generated as a subgroup of* G *by certain subgroups*

$$g_{\mu\nu} G_{\mu\nu} g_{\mu\nu}^{-1}, \quad G_{\mu\nu} \subset G_\mu, \quad g_{\mu\nu} \in \ker f \subset \overline{G}, \textit{ for } \nu \textit{ in an index set } N_\mu,$$

AMS Subject Classifications. Primary 2052, 2095; Secondary 2010.

Key Words and Phrases. Free product of groups with amalgamation, amalgamated subgroup, groupoid, covering groupoid.

[1] These results are contained in the author's doctoral thesis submitted to Princeton University, written under the direction of Professor J. Stallings at the University of California (Berkeley). This research was supported by a Danforth Graduate Fellowship.