DIFFEOMORPHISMS FOR HILBERT MANIFOLDS AND HANDLE DECOMPOSITION

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1. We announce here the following result:

Two homotopic diffeomorphisms of a paracompact separable hilbert manifold of infinite dimension are isotopic.

(1) In this paper, a hilbert manifold (h-manifold) with or without boundary is always hausdorff, paracompact, separable C^∞-differentiable and with the infinite dimensional separable hilbert space H as local model.

Let $M(M, \partial M)$ be an h-manifold (with boundary), $X(X, \partial X)$, an h-manifold or finite dimensional manifold (with boundary).

(a) A closed imbedding $\phi: X \to M(\phi:(X, \partial X) \to (M, \partial M))$ is a C^∞-injective map $\phi: X \to M$, such that the differential $d_* \phi(x)$ is injective for any x, and $\phi(M)$ is closed (for the case with boundary we ask more, $\phi^{-1}(\partial M) = \partial M$ and ϕ is transversal to ∂M in ∂M).

(b) A closed tubular neighborhood of a closed imbedding of infinite codimension, $\phi: X \to M$, $(\phi:(X, \partial X) \to (M, \partial M))$, is a closed imbedding $\tilde{\phi}: X \times D^\infty \to M(\tilde{\phi}:(X, \partial X) \times D^\infty \to (M, \partial M))$ which extends to an open imbedding $\tilde{\Phi}: X \times H \to M(\tilde{\Phi}:(X, \partial X) \times H \to (M, \partial M)$ with $\tilde{\Phi}^{-1}(\partial M) = \partial X \times H)$.

REMARKS. (1) Any closed imbedding of infinite codimension has closed tubular neighborhoods [**3**].

(2) For $\tilde{\phi}_1$ and $\tilde{\phi}_2$ two closed tubular neighborhoods of a closed imbedding $\phi: X \to M(\phi:(X, \partial X) \to (M, \partial M))$, there exists an isotopy $h_t: M \to M$, $(h_t:(M, \partial M) \to (M, \partial M))$, $0 \leqq t \leqq 1$, such that $h_0 = \mathrm{id}$, $h_t \cdot \phi = \phi$ and $h_1 \cdot \tilde{\phi}_1 = \tilde{\phi}_2$ [**2**, Theorem 4.1]. By an isotopy as in [**2**], we mean a level preserving C^∞-diffeomorphism $h: M \times I \to M \times I$, $(h:(M, \partial M) \times I \to (M, \partial M) \times I)$, i.e., $h(x, t) = (h_t(x), t)$.

(c) Let $M(M, \partial M)$ be an h-manifold (with boundary); A closed imbedded submanifold with boundary $(A, \partial A)$, such that $A \subset \operatorname{Int} M$ and $A \backslash \partial A$ is open submanifold of M, is called a zero-codimensional closed submanifold (0-c-submanifold).

The 0-c-submanifold $(B, \partial B)$ is called a collar neighborhood of the 0-c-submanifold $(A, \partial A)$, if $A \subset \operatorname{Int} B$ and $(B \backslash \operatorname{Int} A, \partial(B \backslash \operatorname{Int} A))$ is diffeomorphic to $(\partial A \times [0, 1], \partial A \times \partial [0, 1])$.

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