# ON RANDOM SEQUENCES OF INTEGERS[1]

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A "single transit" method is given for the computer-production of random sequences of integers with prescribed expected density, employing a "Poisson series of trials", which is faster and more flexible than "sieve devices". The mathematical probability theory underlying the method is developed, and some special cases are studied with regard to gap distribution and the "Goldbach property". We state here, without proof, the principal results; the details will be found in a forthcoming Los Alamos report (LA-4268).

1. **The Poisson series of trials.** Let $\{P_n;\ n=1,\ 2,\cdots\}$ be a sequence of numbers $0<P_n\leqq 1$, such that $E(N)\equiv\sum_1^N P_n\to\infty$. The corresponding Poisson series of trials consists in setting $\nu_n=1$ or $0$ with probability $P_n$ or $Q_n\equiv 1-P_n$, respectively, at the same time accepting or rejecting $n$ as a member of the random sequence $B$. $B(N)=\nu_1+\cdots+\nu_N$, $N\geqq 1$, thus counts the number of $n\leqq N$ in $B$, and the law of large numbers insures that the probability

$$(1)\qquad P\{\,|\,B(N)-E(N)\,|\,<\rho E(N)\}\geqq 1-1/\rho^2E(N)\to 1$$

where $\rho$ is any preassigned relative error.

If $F(N)$ is any function asymptotic to $E(N)$, we have equally well

$$(2)\qquad P\{\,|\,B(N)-F(N)\,|\,<3\rho F(N)\}\geqq 1-1/\rho^2(1-\rho)F(N)\to 1.$$

2. **Two practical devices.** In practice, we are *given* a distribution function $F(N)$ and require a sequence $P(n)$ producing sequences satisfying (2). This is easily done if $F(N)$ is the value at $x=N\geqq i$ of a positive function $F(x)$ such that

(A) $F(x)\to\infty$; $F'(x)$ exists, is continuous, nonincreasing, with $0<F'(x)\leqq 1$ for $x\geqq i$. For then $P(n)\equiv F'(n+i-1)$, $n\geqq 1$, gives the desired result.

As an example, we note that $P(n)=2/\log\ (2n+7)$ generates random sequences $\pi$ of odd integers ($2n+1\in\pi$ iff $n\in B$) in the prime-like (expected) distribution $F(2N+1)=\int_2^{2N+1}d\xi/\log\ \xi$. In similar fashion, $P(n)=3/\log\ (3n+20)$ produces sequences $\pi'$ of odd integers

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