SPECTRAL DECOMPOSITION OF ERGODIC FLOWS ON L^p[1]

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Let M be a totally σ-finite measure space and U_s (s real) be a one parameter group of measure—preserving transformations of M satisfying appropriate measurability and continuity conditions. We let $U_s: L^p(M) \to L^p(M)$ by $U_s f = f U_s$. If $p=2$ Stone's spectral theorem for unitary operators [2] says that there is a spectral family of projections $E_\lambda: L^2(M) \to L^2(M)$ such that for $f \in L^2(M)$

$$U_s f = \int_{-\infty}^{\infty} e^{2\pi i \lambda s} dE_\lambda f \tag{1}$$

from which we show that if $\psi \in L^1(R)$ and $\hat{\psi}$ is the Fourier transform of ψ,

$$\int_{-\infty}^{\infty} \hat{\psi}(\lambda) dE_\lambda f = \int_{-\infty}^{\infty} \psi(s) U_s f ds. \tag{2}$$

We will say that a function is normalized at its jumps if it has only jump discontinuities and the value at each jump is the average of the values from the sides. Let χ_τ be the normalized characteristic function of $(-\infty, \tau]$. We approximate χ_τ pointwise with the Fourier transforms of L^1 functions and use (2) to show for $f \in L^2(M)$, $D_\lambda f = E_{\lambda-0} f + E_\lambda f - f$ where

$$D_\lambda f = \frac{-1}{i\pi} \text{ p.v.} \int_{-\infty}^{\infty} \frac{1}{s} e^{-2\pi i \lambda s} U_s f ds \tag{3}$$

and so $E_\lambda f = f + \frac{1}{2} D_\lambda f - \frac{1}{2} D_\lambda^2 f$.

A slight modification of a theorem in [1] shows that D_λ is a bounded transformation on $L^p(M)$ $(1 < p < \infty)$ with the bound independent of λ. This gives

THEOREM 1. *D_λ and hence E_λ extend from $L^p(M) \cap L^2(M)$ to $L^p(M)$ by continuity. For $f \in L^p(M)$, $\|E_\lambda f\|_p$ is bounded uniformly in λ. $E_{\lambda+0} f = E_\lambda f$. $E_\lambda E_\tau f = E_\lambda f$ if $\lambda \leqq \tau$. $\|E_\lambda f\|_p \to 0$ as $\lambda \to -\infty$. $\|E_\lambda f - f\|_p \to 0$ as $\lambda \to +\infty$.*

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