A NECESSARY AND SUFFICIENT CONDITION FOR ORDERS IN DIRECT SUMS OF COMPLETE SKEWFIELDS TO HAVE ONLY FINITELY MANY NONISOMORPHIC INDECOMPOSABLE INTEGRAL REPRESENTATIONS

BY K. W. ROGGENKAMP

Communicated by Irving Reiner, May 15, 1969

Let K be an algebraic number field with ring of integers R. For an R-order Λ in the semisimple K-algebra A it seems to be one of the most important problems—from the viewpoint of integral representations—to characterize those orders Λ, for which the number $n(\Lambda)$ of nonisomorphic indecomposable Λ-lattices is finite. This problem is far from having a satisfying solution. However, a breakthrough came at the end of 1967, when Drozd-Roiter [3] and Jakobinski [5] gave, independently of each other, a necessary and sufficient condition for the finiteness of $n(\Lambda)$, in case Λ is commutative. Whereas Jakobinski's methods seem to be restricted to the commutative case, the methods of Drozd-Roiter bear the possibilities of a generalization to the noncommutative case. This note shall be a small contribution in that direction: We shall give here a necessary and sufficient condition for the finiteness of $n(\Lambda)$ in case Λ is an order in a direct sum of skewfields over a $\mathcal{P}$-adic number field. We shall first fix the notation and then sketch the proof of our theorem; a more explicit version is going to be published later (cf. [6], [7]).

R: a complete discrete rank one valuation ring with finite residue class field,

K: the quotient field of R,

D_i: $1 \leqq i \leqq n$: finite dimensional separable skewfields over K, $A = \sum_i^n \oplus D_i$,

Γ: the unique maximal R-order in A,

Λ: an R-order in A,

$N = \text{rad}\,(\Lambda)$: the Jacobson radical of Λ,

${}_\Lambda\mathfrak{M}^f$: the category of finitely generated unitary left Λ-modules,

${}_\Lambda\mathcal{P}^f$: the category of the projective modules in ${}_\Lambda\mathfrak{M}^f$,

${}_\Lambda\mathfrak{M}^0$: the category of Λ-lattices; i.e., $M \in {}_\Lambda\mathfrak{M}^f$ with $M \in {}_R\mathcal{P}^f$,

$n(\Lambda)$: the number of nonisomorphic indecomposable Λ-lattices,

$\mu_\Lambda(X)$: the minimal number of generators of $X \in {}_\Lambda M^f$,

$\text{rad}_\Lambda(X)$: the intersection of the maximal left Λ-submodules of $X \in {}_\Lambda\mathfrak{M}^f$.