ONE DIMENSIONAL WITT'S THEOREM OVER MODULAR LATTICES[1]

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We first present the problem in a general setting. Let R be a commutative ring with unity. A *quadratic R-space*, in the sense of Bass [1], is a pair (P, q), where P is a finitely generated projective R-module and $q: P \to R$ is a nonsingular quadratic form. An element $x \in P$ is *unimodular* if its coefficient ideal $o_P(x) = \{f(x) \mid f \in \mathrm{Hom}_R(P, R)\} = R$. The orthogonal group $O(P)$ on (P, q) is the set of all R-automorphisms of P preserving the quadratic structure. The one-dimensional Witt's Theorem is concerned with finding the necessary and sufficient conditions under which $O(P)$ acts transitively on the unimodular elements of (P, q).

A. Roy [6] showed that with finiteness assumption on R and if 2 is unitary in R and, if further, the hyperbolic dimension on P is large enough, then $O(P)$ acts transitively on the nonsingular elements of P of a given norm. (A nonsingular element $x \in (P, q)$ is one which has norm $q(x)$ equaling to a unit.) In this paper, we do not assume the element 2 is a unit. However, we strongly restrict the nature of the ring R. Our ring R always denotes the ring of integers in a *local field* F. By a *local field* F, we shall mean here that F is either

(i) a finite extension of the p-adic number field Q_p, for any prime p, or in the characteristic two situation,

(ii) the field of formal power series in one uniformizing variable π over a finite field of constants having characteristic 2.

For such a ring R, a quadratic R-space is a free R-module by Nakayama, and we shall call the pair (P, q) then a *(uni-) modular quadratic R-lattice*. Given an unimodular (or *maximal*) element $z \in P$, the *characteristic set* $\mathfrak{M}_z$ of z in P is defined as

$$\mathfrak{M}_z = \{x \in P \mid B_q(z, x) = 1\},$$

where

$$\begin{aligned} q(x + y) - q(x) - q(y) &= 2B_q(x, y) \qquad \mathrm{Char}(R) \neq 2, \\ &= B_q(x, y) \qquad \mathrm{Char}(R) = 2. \end{aligned}$$

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