CONDITIONS FOR EQUALITY OF THE MACKEY AND STRICT TOPOLOGIES

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1. **Introduction.** In 1958 R. C. Buck [2] made a study of the β or strict topology (introduced in [1] and named for its resemblance to a topology of Beurling) on the space $C(S)$ of bounded continuous functions on a locally compact space S under which the dual of $C(S)_\beta$ is the space $M(S)$ of bounded regular Borel measures on S. In 1966, J. B. Conway [4], [5] showed that when S is σ-compact (or even paracompact) the strict topology is the Mackey topology on $C(S)_\beta$ —the finest locally convex topology on $C(S)$ for which the dual is $M(S)$. However, when S is the space of ordinals $[1, \Omega)$ less than the first uncountable ordinal Ω, Conway showed that β is not the Mackey topology on $C(S)_\beta$.

In [10] Wang studied the strict topology generalized to Banach algebras. More recently, the author and D. C. Taylor [8] studied the strict topology defined by a Banach algebra B with approximate identity on a left Banach B-module X by way of the seminorms $x \to \|Tx\|$, one for each $T \in B$ such that B separates points of X. No necessary or sufficient conditions were obtained for which this general strict topology β is the Mackey topology on X_β. In this paper, sufficient conditions are given in order that β be the Mackey topology on X_β, with our aim being to obtain conditions which in some sense differentiate between the case where S is σ-compact, as opposed to $S=[1, \Omega)$, but in the general setting of [8]. A crucial step in the argument is provided by some results which generalize that of Dorroh [6] and show that the continuity of linear maps on X_β is often determined by their continuity on norm bounded sets in X.

2. **Some observations.** It is a known result that if a space E has its Mackey topology $\tau=\tau(E, E')$, then every continuous map on E into a locally convex space F with its weak topology is continuous on E into F with its given topology. Actually one can prove

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