ON BIEBERBACH EILENBERG FUNCTIONS

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I. **Introduction.** In this paper we bring the following two results: *Suppose that* $F(z)=b_1z+b_2z^2+, \cdots$ *is a B.E. function* (*i.e.* $F(z)$ *is regular in the unit circle,* $F(z)F(\zeta)\neq 1$ *for any* $|z|, |\zeta|<1$ *and* $F(0)=0$). *Then we have*

$$\sum_{k=1}^{\infty} |b_k|^2 \leqq 1. \tag{1}$$

This result contains, of course, the result

$$|b_n| \leqq 1, \qquad n = 1, 2, \cdots \tag{2}$$

which was conjectured by Rogosinsky [8] and was solved about ten years later by Lebedev and Milin [5].

The second result deals with univalent B.E. function $F(z) = b_1z+b_2z^2+ \cdots$. For such function we have the following

$$|b_n| \leqq e^{-c/2}(n-1)^{-1/2}, \qquad n = 2, 3, \cdots, \tag{3}$$

where c is Euler constant.

This result is sharp in order of magnitude and the constant cannot be improved to be better than $e^{-1/2}$.

II. **The results of Lebedev and Milin.** Lebedev and Milin found [6], [7] some important results concerning coefficients of exponential functions which we quote here.

LEMMA 1. *Let* $A_1, A_2, A_3, \cdots$ *be an infinite sequence of arbitrary complex numbers such that* $\sum_{k=1}^{\infty} k|A_k|^2<\infty$. *Then for* $\exp \sum_{k=1}^{\infty} A_kz^k = \sum_{k=0}^{\infty} D_kz^k$ *we have*

$$\sum_{k=0}^{\infty} |D_k|^2 \leqq \exp \sum_{k=1}^{\infty} k|A_k|^2 \tag{4}$$

with equality only in the case $A_k=\rho^k\eta^k/k$, $k=1, 2, \cdots$ *where* $0\leqq\rho<1$ $|\eta|=1$.

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