HOMOLOGICAL PROPERTIES OF THE RING OF DIFFERENTIAL POLYNOMIALS[1]

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The ring of differential polynomials over a universal differential field (Kolchin [7]), and the ring of twisted polynomials $\overline{F}_2[t, \rho]$, where $\overline{F}_2$ is an algebraic closure of $Z/2Z$ and ρ is the automorphism of $\overline{F}_2$ defined by: $z \to z^2$, "localized" at the multiplicative subset $\{t^k \mid k \text{ an integer} \geqq 0\}$, provide examples of a principal right and left ideal domain R, not a field, that is a right V-ring (i.e., each simple right R-module is injective). Such a ring was conjectured to exist by Carl Faith. Both examples are shown to have a unique simple right R-module. If R is either example, then by definition of a right V-ring, every right R-module has a maximal submodule. Bass proved that if a ring A satisfies the d.c.c. on principal left ideals, then A has a bounded number of orthogonal idempotents and every right A-module has a maximal submodule. The above examples show that the converse is false, thus answering a question raised by Bass [1, p. 470].

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1. **Differential polynomials and right V-rings.** Throughout this paper each ring R will be a ring with an identity element 1, and each right R-module M will be unitary in the sense that $x1 = x$ for all $x \in M$. Mod-R will denote the category of all right R-modules.

DEFINITION 1. A ring R is a *right V-ring* (after Villamayor) in case the following equivalent conditions are satisfied:

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