ELEMENTARY CYCLES OF FLOWS ON MANIFOLDS

BY M. C. IRWIN

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There are two very natural notions of equivalence of flows (see [1], [2]) on a manifold. One is the existence of a homeomorphism mapping orbits onto orbits, preserving the natural orientation of orbits but not necessarily their natural parametrisation. The second requires that the homeomorphism alter natural parametrisations by at most a positive constant multiple. We call the first relation on flows *orbit-equivalence* and the second *flow-equivalence.* There are obvious localisations of these relations. In general flow-equivalence is strictly stronger than orbit-equivalence. However, it is a consequence of the theorem of Hartman [5], [6], [7] and Grobman [3], [4] that the local notions of equivalence are the same at elementary (see [1], [2]) rest-points. The purpose of this note is to announce a similar result for elementary cycles. M. Shub has informed me that he and C. Pugh have also obtained this result.

1. **Preliminaries.** Let $\phi: \boldsymbol{R}\times X\to X$ be a C^1 flow on a C^∞ manifold X. We write $\phi_x(t)=\phi^t(x)=\phi(t, x)$, so that, for fixed $x\in X$, $\phi_x: \boldsymbol{R}\to X$ is C^1 and, for fixed $t\in \boldsymbol{R}$, $\phi^t: X\to X$ is a C^1 diffeomorphism. Let U be open in X. For fixed $x\in U$ let I_x denote the component of $(\phi_x)^{-1}(U)$ containing 0, and let D_U denote $\bigcup_{x\in U} I_x\times\{x\}$.

Now suppose that Ψ is a C^1 flow on a C^∞ manifold Y and that A and B are subsets of X and Y respectively. We say that A is *flow-equivalent* to B (with respect to the given flows) if there exist open neighbourhoods U of A and V of B and a homeomorphism $h: U\to V$ such that $h(A)=B$ and, for all $(t, x)\in D_U$,

$$h\phi(t, x) = \Psi(\alpha(t), h(x)),$$

where $\alpha: \boldsymbol{R}\to\boldsymbol{R}$ is a multiplication by some positive constant. In this case h maps orbit components of ϕ in U onto orbit components of Ψ in V, preserving orientation.

Let $\nu\in GL(\boldsymbol{E})$, where $\boldsymbol{E}$ is a finite dimensional real normed linear space. Let $\boldsymbol{F}$ be the largest invariant subspace of $\boldsymbol{E}$ on which ν has no complex eigenvalues of modulus 1. We call $\nu|\boldsymbol{F}$ the *hyperbolic part* of ν.

Recall [8] that we may associate with any hyperbolic linear auto-