# RECENT RESULTS IN THE FIXED POINT THEORY OF CONTINUOUS MAPS

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1. **Introduction.** Given a function $f$: $X \to X$, any question which inquires into the existence, nature and number of points $x \in X$ such that $f(x) = x$ is called fixed point theory. The assumptions on $f$ and $X$ range from practically none (e.g., $X$ is a set, $f$ a function) to quite stringent assumptions on $f$ and $X$ (e.g., $X$ is a Riemannian manifold and $f$ is an isometry). Our attention will be focused on results which require $X$ to be a fairly reasonable space (e.g., a finite polyhedron) and $f$ a map (=continuous function). Furthermore, we will limit our discussion to results which are not included in the expository tract [**49**] by Van der Walt (1967), which adequately covers the history of the subject from its beginning around 1910 to the early sixties.

2. **The Lefschetz theorem and local index theory.** One of the most useful tools in fixed point theory is the Lefschetz Fixed Point Theorem [**34**], [**35**], [**25**]. In its most elementary form it is simply this. Let $X$ denote a finite polyhedron and $f: X \to X$ a map. Then, using the field of rationals $Q$ as coefficients, $f$ induces homomorphisms.

$$f_{*k}: H_k(X; Q) \to H_k(X; Q). \tag{1}$$

The number (it turns out to be an integer)

$$L(f) = \sum_k (-1)^k \operatorname{Trace} f_{*k} \tag{2}$$

is called the Lefschetz number of $f$. Then a sufficient condition for $f$ to have at least one fixed point is that $L(f) \neq 0$. In short,

$$L(f) \neq 0 \Rightarrow f(x) = x \qquad \text{for some } x \in X. \tag{3}$$

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