QUASI-SUBORDINATION AND COEFFICIENT CONJECTURES[1]

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Abstract. The concepts of subordination and majorization of two analytic functions are unified by regarding them as special cases of the idea of quasi-subordination. Six conjectures are discussed in connection with quasi-subordination, univalent and multivalent functions. Evidence is given to support the truth of these conjectures.

1. **Subordination.** Let D be a simply connected domain on the z-sphere and let $w=F(z)$ be meromorphic on D and map D onto $D(F)$ the Riemann domain over the w-sphere.

Let $w=f(z)$ be also meromorphic in D.

DEFINITION. $f(z)$ is called *subordinate* to $F(z)$ in D, with center z_0 in D, if $f(z_0)=F(z_0)$ and the values of $f(z)$ in D, determined by analytical continuation from z_0, are situated in $D(F)$.

The Riemann domain $D(f)$ is extended (but is not necessarily schlicht) over $D(F)$.

We write $f(z)\prec F(z)$ in D.

There is no loss in generality in assuming D to be the unit disc $E\{z\colon |z|<1\}$ and $z_0=0$. Under these assumptions there exists a function $w(z)$ regular in $|z|<1$ with $|w(z)|\leqq|z|<1$ such that $f(z)=F\{w(z)\}$ $(|z|<1)$.

In our present discussion we shall be concerned with the case that $f(z)$ and $F(z)$ are both regular in F. Frequently we shall take $f(0)=F(0)=0$.

2. **Majorization.** If $f(z)$ and $F(z)$ are both regular in E, and if

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