DUALITY THEORY FOR GROTHENDIECK CATEGORIES[1]

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In [3] Jan-Eric Roos has shown that an abelian category $\mathfrak{A}$ is a locally Noetherian Grothendieck category if and only if (iff) it is dual to the category of complete (and Hausdorff) topologically coherent left modules over some complete topologically left coherent and left coperfect ring R. In this note I announce a series of results which show similar relations for arbitrary, not necessarily locally Noetherian, Grothendieck categories.

Some preliminary notions are needed. A *Grothendieck category* is an abelian cocomplete category with exact filtered colimits and a family of generators. The dual of a Grothendieck category is a *co-Grothendieck category*. A topological ring R is a ring (always unital) with a topology such that the addition and multiplication are continuous. A topological ring R is called *left linear topological* [1, p. 411] if 0 has a basis of neighborhoods consisting of left ideals. Such a basis of neighborhoods is simply called a basis of R. Given a topological ring R, a topological R-left module is an R-left module with a topology such that the addition on M and the multiplication $R\times M\to M$ are continuous. A topological R-left module is called left linear topological if it admits a basis (of neighborhoods of 0) consisting of left submodules. Let R be a left linear topological ring, and let $\mathrm{Dis}_R R$ be the category of discrete topological R-left modules. Then $\mathrm{Dis}_R R$ is a full subcategory of $\mathrm{Mod}_R R$, the category of all R-left modules, and consists of all R-left modules M such that for all $m\in M$ the left annihilator $(0:m)$ is open in R. (Compare [3, §4, Proposition 3]). The category $\mathrm{Dis}_R R$ is the closed subcategory of $\mathrm{Mod}_R R$ associated with the given left linear topology on R [1, p. 411 ff]. Let Coh $(\mathrm{Dis}_R R)$ denote the category of all coherent objects of $\mathrm{Dis}_R R$ [3, §5]; $\mathrm{Coh}(\mathrm{Dis}_R R)$ is a full, skeletal-small subcategory of $\mathrm{Mod}_R R$ and closed under finite limits and colimits (in $\mathrm{Mod}_R R$). In particular, $\mathrm{Coh}(\mathrm{Dis}_R R)$ is abelian. The ring R is called *topologically left coherent* if $\mathrm{Dis}_R R$ admits a family of coherent generators, i.e. if $\mathrm{Coh}(\mathrm{Dis}_R R)$ is a class of generators for $\mathrm{Dis}_R R$ [3, §4, Definition 3].

Let R be any ring. An R-left module M is called *algebraically linearly compact* if it satisfies the following condition: If I is a directed ordered set and if $(M_i;\, i\in I)$ is a decreasing family of finitely gene-

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