A NOTE ON MATRIX SUMMABILITY OF A CLASS OF FOURIER SERIES

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1. Recently several papers by Rajagopal [7], Varshney [11] and others have been written, on Nörlund summability of Fourier series, in order to unify some of the classical results. Though lower semitriangular matrix (Λ) summability method has been known for quite some time no attempt has yet been made to apply it to Fourier series. The object here is to determine a necessary and sufficient condition for (Λ) summability of Fourier series and to include a wider class of known results.

A Fourier series, of a Lebesgue-integrable function, is said to be summable at a point by triangular matrix method (Λ), defined by Hardy [1], if $\Lambda_{n,k}=0$ for $k>n, \sum \Lambda_{n,k}\to 1$ as $n\to\infty$ and $\sum_{k=0}^{n}|\Lambda_{n,k}|\leqq M$, where M is a constant, and the point is in a certain subset of the Lebesgue set.

The following main theorem has been proved here.

THEOREM. *Let a sequence* $\{\Lambda_{n,k}\}$ *be defined in terms of*

$$\text{(1.1)}\qquad \begin{aligned}&\Lambda_n(u), \textit{ monotonic decreasing and strictly positive for all } u \geqq 0,\\ &\Lambda_{n,u} \equiv \Lambda_n(u)\end{aligned}$$

and if

$$\text{(1.2)}\qquad \Phi(t) \equiv \int_0^t |\phi(u)|\,du = o\left(\frac{t}{\psi(1/t)}\right) \quad \textit{as } t\to +0$$

and $\psi(t)$ *be positive, nondecreasing with* t; *then a necessary and sufficient condition for* (Λ) *summability of Fourier series, to* 0 *or*

$$\text{(1.3)}\qquad t_n \equiv \left\{\sum_{k=0}^{n} \Lambda_{n,k}S_k\right\} \to 0 \quad \textit{as } n\to\infty,$$

is

$$\text{(1.4)}\qquad \int_1^n \frac{\bar{\Lambda}_n(u)}{u\psi(u)}\,du = O(1)$$

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