COBORDISM OF REGULAR $O(n)$-MANIFOLDS

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A C^∞ manifold M together with a C^∞ action of $O(n)$ on M is said to be a regular $O(n)$-manifold if, for each $m \in M$, the isotropy group of m, $O(n)_m = \{g \in O(n) \mid gm = m\}$, is conjugate in $O(n)$ to $O(p)$ for some $p \leqq n$; $O(p)$ is understood to be imbedded in $O(n)$ in the standard way [3]. Compact regular $O(n)$-manifolds M_1^s, M_2^s are said to be (regularly) cobordant if there exists a compact regular $O(n)$-manifold W^{s+1} with ∂W^{s+1} equivariantly diffeomorphic to $M_1 \cup M_2$.

The set of cobordism classes of regular $O(n)$-manifolds of dimension s will be denoted by $\mathfrak{N}O(n)_s$. $\mathfrak{N}O(n)_*$ is a graded algebra over $\mathfrak{N}_*$, the cobordism ring of unoriented manifolds; addition is given by disjoint union, multiplication by cartesian product (with the diagonal action $g(m_1, m_2) = (gm_1, gm_2)$, $(m_1, m_2) \in M_1 \times M_2$) and $\mathfrak{N}_*$ acts by cartesian product (with the obvious action $g(m_1, m_2) = (m_1, gm_2)$, $(m_1, m_2) \in M_1 \times M_2$, $[M_1] \in \mathfrak{N}_*$, $[M_2] \in \mathfrak{N}O(n)_*$).

EXAMPLES. (A) Let M = point. Then $[M] \in \mathfrak{N}O(n)$. The submodule of $\mathfrak{N}O(n)$ (as a $\mathfrak{N}_*$ module) generated by $[M]$ [i.e. trivial $O(n)$ manifolds] is isomorphic to $\mathfrak{N}_*$ and we clearly have a decomposition $\mathfrak{N}O(n)_* = \mathfrak{N}_* \oplus \tilde{\mathfrak{N}}O(n)_*$.

(B) Any manifold with a differentiable involution is a regular $O(1)$ manifold.

(C) If M is a regular $O(n)$ manifold then by restricting the action to $O(n-1) \subset O(n)$ we get a regular $O(n-1)$ manifold. Since restriction respects cobordism there is an $\mathfrak{N}_*$ map $\rho: \mathfrak{N}O(n)_* \to \mathfrak{N}O(n-1)_*$.

(D) Given a regular $O(n)$ manifold M, one can extend the action to a regular $O(n+1)$ action on $O(n+1) \times_{O(n)} M$ and hence there is an $\mathfrak{N}_*$ map ext: $\mathfrak{N}O(n)_s \to \mathfrak{N}O(n+1)_{s+n}$.

(E) Let M be a regular $O(1)$ manifold and let P be an $O(n-1)$ principal bundle. Then $P \times M$ is an $O(n-1) \times O(1)$ manifold and $O(n) \times_{O(n-1) \times O(1)} P \times M$ is a regular $O(n)$ manifold. Hence, there is a homomorphism $h: \mathfrak{N}O(1) \otimes_{\mathfrak{N}_*} \mathfrak{N}_*(BO(n-1)) \to \mathfrak{N}O(n)_*$.

THEOREM. (i) *$\mathfrak{N}O(n)_*$ is a free $\mathfrak{N}_*$ module on countably many generators:*

(ii) *the algebra structure is given by $xy = 0$ for $x, y \in \tilde{\mathfrak{N}}O(n)_*$, $n > 1$,*

(iii) *$\rho \mid \tilde{\mathfrak{N}}O(n)_*$ is the zero map,*

(iv) *ext $\mid \tilde{\mathfrak{N}}O(n)_*$ is a monomorphism onto a direct summand of $\mathfrak{N}O(n+1)_*$; ext $\mid \mathfrak{N}_*$ is zero,*

(v) *h is an epimorphism.*