ON THE EQUATIONS $u_t+\nabla\cdot F(u)+0$ AND $u_t+\nabla\cdot F(u)=\nu\Delta u$[1]

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This paper presents several results on global solutions of the initial value problems for the first order nonlinear conservation law

$$(1) \qquad u_t + \nabla\cdot F(u) = 0$$

and the associated second order nonlinear parabolic equation

$$(2) \qquad u_t + \nabla\cdot F(u) = \nu\Delta u, \qquad \nu > 0$$

for an unknown scalar function $u=u(t, x)$ on the domain $D=\{(t, x)\in R^{d+1};\ t>0\}$. Here $F\in C^\infty(R^1, R^d)$. For both equations, the given initial data are

$$(3) \qquad u(0, x) = u_0(x), \qquad x \in R^d.$$

We call these initial value problems IVP_1 and IVP_2 respectively. They are of interest as simplified prototypes of the initial value problems of gas dynamics (nonviscous and viscous respectively—cf. [2]).

We deal with weak solutions of IVP_1 and IVP_2. If $u\in L_1^{\mathrm{loc}}(D)$, we say that u is a *weak solution of IVP_1* if for each $\phi\in C^1(R^{d+1})$ of compact support

$$(4) \qquad \iint_D [u\phi_t + F(u)\cdot\nabla\phi]\,dxdt + \int_{R^d} u_0(x)\phi(0, x)\,dx = 0.$$

We say that u is a *weak solution of IVP_2* if for each $\phi\in C^2(R^{d+1})$ of compact support

$$(5) \qquad \iint_D [u\phi_t + \nu u\Delta\phi + F(u)\cdot\nabla\phi]\,dxdt + \int_{R^d} u_0(x)\phi(0, x)\,dx = 0.$$

It is well known [2] that weak solutions of IVP_1 are discontinuous and nonunique. For solutions of bounded variation locally in D, Vol'pert [3] has given a supplementary condition, called an *entropy condition*, on the discontinuities of a solution which singles out a unique solution in this class. We call this the *entropy solution*; it exists whenever u_0 is bounded and has bounded variation locally in R^d [3].

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