ON A CONJECTURE OF G. D. MOSTOW AND THE STRUCTURE OF SOLVMANIFOLDS

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Communicated by Nathan Jacobson, June 10, 1969

Introduction. Let G be a connected solvable Lie group and let Γ be a closed subgroup of G. Then the quotient manifold G/Γ is called a solvmanifold. G. D. Mostow in a fundamental paper [6] proved

THEOREM 1. *Let G/C be a compact solvmanifold, let N be the nil-radical of G, and let Γ contain no nontrivial, connected subgroup normal in G. Then*

(a) *N contains the identity component of Γ,*

(b) *$N/N\cap\Gamma$ is compact,*

(c) *$N\Gamma$, the group generated by N and Γ in G, is closed, in G.*

Mostow has also conjectured the following:

MOSTOW CONJECTURE. A solvmanifold is a vector bundle over a compact solvmanifold.

In this paper we will announce results that yield a new proof of Theorem 1 and a proof of the Mostow Conjecture, as well as many of the known results on the structure of solvmanifolds as given in [1], [3] and [4] for instance. An outline of the proof of the Mostow Conjecture and the proof of Theorem 1 are given in §3.

1. **Definitions and resume of known facts.** Let N be a connected, simply connected nilpotent Lie group. A closed subgroup of N will be called a CN group. According to Malcev a CN group Δ can be characterized as a torsion free nilpotent group such that if Δ_0 is the identity component of Δ then Δ/Δ_0 is finitely generated. Further, if Δ is a CN group there exists a unique connected nilpotent Lie group Δ_R such that $\Delta_R\supset\Delta$ and Δ_R/Δ is compact. If Δ is a CN group with Δ_0 trivial we will call Δ an FN group.

In [3] and [6] it was shown that a group Γ is the fundamental group of a compact solvmanifold if and only if Γ satisfies an exact sequence

$$(1)\qquad 1\to\Delta\to\Gamma\to Z^s\to 1$$

where Δ is an FN group and Z^s denotes s copies of the integers. Fundamental groups of compact solvmanifolds will be called FS groups. If Δ in (1) is a CN group we will call Γ a CS group. If Γ is a CS group satisfying the exact sequence (1) there is a unique group Γ_R satisfying the exact diagram: