ON THE FORMAL GROUP LAWS OF UNORIENTED AND COMPLEX COBORDISM THEORY

BY DANIEL QUILLEN[1]

Communicated by Frank Peterson, May 16, 1969

In this note we outline a connection between the generalized cohomology theories of unoriented cobordism and (weakly-) complex cobordism and the theory of formal commutative groups of one variable [4], [5]. This connection allows us to apply Cartier's theory of typical group laws to obtain an explicit decomposition of complex cobordism theory localized at a prime p into a sum of Brown-Peterson cohomology theories [1] and to determine the algebra of cohomology operations in the latter theory.

1. **Formal group laws.** If R is a commutative ring with unit, then by a *formal* (commutative) *group law* over R one means a power series $F(X, Y)$ with coefficients in R such that

(i) $F(X, 0)=F(0, X)=X$,

(ii) $F(F(X, Y), Z)=F(X, F(Y, Z))$,

(iii) $F(X, Y)=F(Y, X)$. We let $I(X)$ be the "inverse" series satisfying $F(X, I(X))=0$ and let

$$\omega(X) = dX/F_2(X, 0)$$

be the normalized invariant differential form, where the subscript 2 denotes differentiation with respect to the second variable. Over $R\otimes \mathbf{Q}$, there is a unique power series $l(X)$ with leading term X such that

$$l(F(X, Y)) = l(X) + l(Y). \tag{1}$$

The series $l(X)$ is called the *logarithm* of F and is determined by the equations

$$\begin{aligned} l'(X)dX &= \omega(X), \\ l(0) &= 0. \end{aligned} \tag{2}$$

2. **The formal group law of complex cobordism theory.** By *complex cobordism theory* $\Omega^*(X)$ we mean the generalized cohomology theory associated to the spectrum MU. If E is a complex vector bundle of dimension n over a space X, we let $c_i^{\Omega}(E)\in\Omega^{2i}(X)$, $1\leqq i\leqq n$ be

[1] Alfred P. Sloan Foundation Fellow. This work was supported also by NSF GP9006.