MAXIMAL RATES OF DECAY OF SOLUTIONS OF PARTIAL DIFFERENTIAL EQUATIONS

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It has been proved by C. Morawetz [2] that if $u(x, t)$ is a solution of the relativistic wave equation

$$u_{tt} - \Delta u + u = 0$$

for all $x = (x_1, x_2, \cdots, x_n)$ and t, having finite energy at $t=0$, and vanishing in the forward light cone $|x| < t$, $t > 0$, then it must vanish identically. On the other hand the author [1] has obtained a generalization of Rellich's Theorem (concerning decay of solutions of the reduced wave equation $\Delta u + u = 0$) to a class of (not necessarily elliptic) equations with constant coefficients of arbitrary order. The present note is intended to announce a number of results which are natural generalizations of and improvements of both aforementioned results. Detailed proofs will appear elsewhere.

Let $P(\xi) = P(\xi_1, \xi_2, \cdots, \xi_N)$ be a polynomial with real coefficients. Throughout, we make the following assumptions:

1. The real solution set S of $P(\xi) = 0$ is nonempty.
2. Grad $P(\xi) \neq 0$ in S, and hence S is a smooth $N-1$ dimensional manifold.
3. The Gaussian curvature of S never vanishes.

Assign a unit normal $\boldsymbol{n}$ to each point of S, varying continuously. The totality of all $\boldsymbol{n}$ fill an open set $\mathfrak{N}$ on the unit sphere, giving rise to an open cone $\mathcal{K}$ in R^N in the sense that $\mathcal{K}$ consists of all $r\boldsymbol{n}$, $\boldsymbol{n} \in \mathfrak{N}$, $r \geqq 0$.

Define $\mathfrak{N}_\epsilon$ as that subset of $\mathfrak{N}$ consisting of points whose (spherical) distance to the boundary of $\mathfrak{N}$ exceeds ϵ. $\mathcal{K}_\epsilon$ will denote the cone generated by $\mathfrak{N}_\epsilon$. $-\mathfrak{N}$ will denote the set of vectors $-\boldsymbol{n}$, with $\boldsymbol{n} \in \mathfrak{N}$, and similarly for $-\mathcal{K}$. $\mathfrak{N}'$ denotes the complement of $\mathfrak{N}$ on the unit sphere, and $\mathcal{K}'$ the corresponding cone. $\overline{\mathcal{K}}$ denotes the closure of K.

We will write

$$Lu \equiv P\left(\frac{1}{i}\frac{\partial}{\partial x_1}, \cdots, \frac{1}{i}\frac{\partial}{\partial x_N}\right)u \equiv P\left(\frac{1}{i}\frac{\partial}{\partial x}\right)u.$$

THEOREM I. *Suppose, under the foregoing Assumptions* 1–3, $u(x)$ *is a function satisfying*