UNIFORM ALGEBRAS ON CURVES

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1. **Results.** A recent result of H. S. Shapiro and A. L. Shields [4] states that if f and g are continuous complex valued functions on the unit interval I such that together they separate the points of I and also that f alone separates all but one pair of points, then the closed subalgebra of $C(I)$ generated by f and g is all of $C(I)$. Two generalizations are:

THEOREM. *Let A be a separating uniform algebra on I such that there exists an f in A which is locally* 1-1, *then $A = C(I)$.*

THEOREM. *Let A be a separating uniform algebra on I generated by two functions f and g such that there is a compact totally disconnected subset E of I such that*
(i) *$f|E$ is constant, and*
(ii) *f separates every pair of points of I not both of which are in E. Then $A = C(I)$.*

The proofs use the notion of analytic structure in a maximal ideal space. J. Wermer first obtained results along these lines and further contributions were made by E. Bishop and H. Royden and then by G. Stolzenberg [5] who proved

STOLZENBERG'S THEOREM. *Let $X \subseteq \mathbf{C}^n$ be a polynomially convex set. Let $K \subseteq \mathbf{C}^n$ be a finite union of $\mathcal{C}^1$-curves. Then $(X \cup K)\hat{} - X \cup K$ is a (possibly empty) pure* 1*-dimensional analytic subset of $\mathbf{C}^n - X \cup K$. (See* [5] *for the notation and definitions.)*

A further result of Stolzenberg (and Bishop) is that a $\mathcal{C}^1$ arc $K \subseteq \mathbf{C}^n$ is polynomially convex and $P(K) = C(K)$. It is well known that no smoothness is needed in $\mathbf{C}^1$ but that in higher dimensions further assumptions are required for the above conclusion. We have

THEOREM. *Let $f_1, f_2, \cdots, f_n \in C(I)$ separate the points of I and suppose that for $1 \leqq i \leqq n-1$, f_i is either $\mathcal{C}^1$ or real-valued. Then the separating uniform algebra which $f_1, f_2, \cdots, f_n$ generate is $C(I)$.*

If all the f_i, $1 \leqq i \leqq n-1$ are real valued, this theorem reduces to a result of Rudin [3]; on the other hand, if we consider the image K of I under $t \to (f_1(t), \cdots, f_n(t))$ we obtain a generalization of Stolzenberg's result on smooth arcs.