EMBEDDING SPHERES AND BALLS IN CODIMENSION $\leqq 2$

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1. **Introduction.** In this note we announce some results on existence of PL embeddings of n-spheres and n-balls into a compact $(n-1)$-connected q-manifold ($n \geqq q-2$) by extending techniques of our preceding papers [5], [4]. Details will appear later. The result for locally flat embeddings with codimension two is satisfactory, although in general the low dimensional cases are still open.

By $\bigcup_{k=1}^r D_k^n$, $\bigcup_{k=1}^r S_k^n$ we denote the disjoint unions of r copies of the standard PL n-ball D^n, the standard PL n-sphere $S^n = \partial D^{n+1}$, resp. The embedding theorem of balls in codimension $\leqq 2$ is as follows:

THEOREM A. *Let Q be a compact $(n-1)$-connected PL q-manifold with nonempty boundary ∂Q.*

Let ϕ: $\bigcup_{k=1}^r D_k^n \to Q$ be a map such that $\phi(\bigcup_{k=1}^r S_k^{n-1}) \subset \partial Q$ and $\phi | \bigcup_{k=1}^r S_k^{n-1}$ is a PL embedding.

(I). *Suppose that one of the following holds.*

(0) $q = n \neq 3, 4,$

(1) $q = n+1 \neq 4,$

(2) $q = n+2 \neq 4$ *and* $r = 1.$

Then ϕ is homotopic to a proper PL embedding f: $\bigcup_{k=1}^r D_k^n \to Q$ keeping $\phi | \bigcup_{k=1}^r S_k^{n-1}$ fixed.

(II). *Suppose that $\phi | \bigcup_{k=1}^r S_k^{n-1}$ is locally flat, and that*

(1) $q = n+1 \neq 4$ *or*

(2) $q = n+2 =$ *odd and* $r = 1.$

Then ϕ is homotopic to a locally flat PL embedding f: $\bigcup_{k=1}^r D_k^n \to Q$ keeping $\phi | \bigcup_{k=1}^r S_k^{n-1}$ fixed.

(*Refer to* [13, *Chapter* 8, *Corollary* 5].)

In case $q - n = 0$, Theorem A, (I) is equivalent to the generalized Poincaré conjecture. In case $q = n+1 = 4$, Theorem A is still open. In case $n = 2$ and $Q = D^4$, refer to [13, Chapter 8, Counterexample 1].

In case $q = n+2 =$ even, Theorem A, (II) is false because of the existence of nonslice knots ([1] and [6, Chapter III]).

The embedding theorem of spheres in codimension $\leqq 2$ is as follows:

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