ORDERS IN ARTINIAN RINGS

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The object of this note is to indicate some results concerning (right) skew polynomial rings over right orders in right Artinian rings. Detailed proofs will be published elsewhere.

We recall a few definitions first. A *right Artinian ring* is a ring with unity satisfying the descending chain condition on right ideals. A subring S of a right Artinian ring Q is called a *right order* in Q if every regular element in S is a unit in Q and every $q \in Q$ can be expressed as $q = sc^{-1}$, where $s, c \in S$.

Let R be a ring with unity and S be a unitary subring of R. Suppose there exists an element $x \in R$ such that every nonzero $r \in R$ can be uniquely expressed as

$$r = \sum_{i=0}^{k} x^{n_i} s_i$$

where s_i, $0 \leqq i \leqq k$, are nonzero elements of S and $0 \leqq n_0 < \cdots < n_k$ are integers. Further, suppose there exists a unitary monomorphism $\rho: S \to S$ such that $sx = x\rho(s)$ holds for every $s \in S$. In such a situation, we denote R as $S[x, \rho]$ and call it a right *skew polynomial ring* over S. If I is an ideal of S such that $\rho(I) \subseteq I$ then I is said to be *ρ-invariant*. We have

THEOREM 1. *Let S be a ring with unity which is a right order in a right Artinian ring and let $R = S[x, \rho]$. Then R is a right order in a right Artinian ring $\hat{Q}$. $\hat{Q}$ is semisimple if and only if S is semiprime. $\hat{Q}$ is simple if and only if S is semiprime and every nonzero ρ-invariant two-sided ideal of S is an essential right ideal of S.*

We need some more definitions. Let Q be a semisimple (Artinian) ring, $\{f_1, \cdots, f_m\}$ be the set of all the distinct central idempotents of Q which are primitive in the centre of Q and let $\rho: Q \to Q$ be a monomorphism. Then there exists a unique permutation σ on $\{1, \cdots, m\}$ such that $\rho(f_i) = f_{\sigma(i)}$ for $1 \leqq i \leqq m$. If $\sigma = \sigma_1 \cdots \sigma_k$ is a decomposition of σ into disjoint cycles then $\max_{1 \leq i \leq m}\{\text{length } \sigma_i\}$ is is called the *shuffling index* of ρ. Recall that a ring R has *right rank m* if m is the least integer such that every right ideal of R has a system of generators containing at most m elements.