APPROXIMATING HOMOTOPIES BY ISOTOPIES IN FRÉCHET MANIFOLDS

BY JAMES E. WEST

Communicated by Richard D. Anderson, April 18, 1969

Let M be an F-manifold, that is, a separable, metric manifold modelled on an infinite-dimensional Fréchet space. The question was raised at a problem seminar this January (1969) at Cornell University whether homotopic embeddings of another F-manifold in M are isotopic. In this note the affirmative answer is given and a stronger result established.

Given an open cover $\mathcal{U}$ of a space X, two maps f and g of a space Y into X are said to be *$\mathcal{U}$-close* provided that for each y in Y there is an element of $\mathcal{U}$ containing both $f(y)$ and $g(y)$. The two maps are said to be *pseudo-isotopic* provided there is a map h: $Y\times I\to X$ with

$$h(y, 0) = f(y), \qquad h(y, 1) = g(y)$$

and which for each t in $(0, 1)$ is an embedding of $Y\times\{t\}$. The theorem is as follows:

THEOREM. *Homotopic maps of a separable metric space into an F-manifold are pseudo-isotopic. If the domain is complete, the pseudo-isotopy may be required to be through closed embeddings. Furthermore, given any open cover $\mathcal{U}$ of the manifold and any homotopy F between the maps, the pseudo-isotopy may be required to be $\mathcal{U}$-close to F.*

PROOF. Let X be the separable metric space, M the F-manifold, and f and g the homotopic maps of X into M. By a collection of results, all separable, infinite-dimensional Fréchet spaces are homeomorphic to the countably infinite product s of open intervals $(-1, 1)$. (For a discussion of these results and a bibliography, see the introduction of [3].) Furthermore, a theorem of R. D. Anderson and R. M. Schori [4] asserts that given any open cover $\mathcal{U}$ of M, there is a homeomorphism $h_{\mathcal{U}}$ of M onto $M\times s$ so that $p\circ h_{\mathcal{U}}$ is $\mathcal{U}$-close to the identity map, where p is the projection onto M. If $\{s_i\}_{i=1}^{\infty}$ is a countable, indexed family of copies of s, it is easy to see that s', the product of the s_i's, is homeomorphic to s, so s may be replaced by s' in the above theorem.

For each integer i and real number t in $(-1, 1)$, let $\psi_{i,t}$: $s_i\to s_i$ be the map which multiplies in each coordinate by t, and let