ON A NONTRIVIAL HIGHER EXTENSION OF REPRESENTABLE ABELIAN SHEAVES[1]

BY LAWRENCE S. BREEN

Communicated May 2, 1969

Let $\mathcal{S}$ be the category of abelian sheaves in the *fppf* topology over a base scheme S, as defined in Demazure and Grothendieck [3, exposé IV §6.3]. This is an abelian category with enough injectives (see Artin [1, 1.6, 1.8]). For any F in $\mathcal{S}$ and any integer $i \geqq 0$, the functor $\mathrm{Ext}^i(F, -)$ from $\mathcal{S}$ to the category of abelian groups is defined in the usual manner to be the ith derived functor of the functor $\mathrm{Hom}(F, -)$. Let $S = \mathrm{Spec}(k)$ where k is a separably closed field of characteristic 2; we denote by α_2 the scheme $\mathrm{Spec}\,(k[x]/(x^2))$ with the usual group law (see for example Oort [8]), by G_m the multiplicative group scheme, and identify these objects of the category $\mathcal{C}$ of commutative algebraic group schemes over S with the objects in $\mathcal{S}$ which they represent. We show that $\mathrm{Ext}^2(\alpha_2, G_m) \neq 0$.

Via the identification just mentioned, $\mathcal{C}$ is a full subcategory of $\mathcal{S}$ which however does not contain enough injectives. It is nonetheless possible to define a functor Ext^i within the category $\mathcal{C}$. For $G, G' \in \mathcal{C}$, define $\mathrm{Ext}^i(G, G')$ to be the group of equivalence classes of i-fold Yoneda extensions in $\mathcal{C}$ of G by G'. This point of view, which was introduced by Serre in [9], was systematically developed by Oort in [8]. Oort shows in particular that $\mathrm{Ext}^i(H, G_m) = 0$ for $i \geqq 1$, where H is any finite group scheme over an algebraically closed groundfield. Our computation thus illustrates the fact that the two definitions of Ext^i are not equivalent.

I wish to thank A. K. Bousfield and B. Mazur for their valuable help during the preparation of this work.

The technique used below in computing $\mathrm{Ext}^i(\alpha_2, G_m)$ (where henceforth we will always mean the first definition of Ext^i) is that of [2]; since only a small part of the theory described there is needed in our special case, we restate in detail the facts required.

Eilenberg and MacLane have defined [4, p. 659], [5] for every abelian group G a complex of free abelian groups $A(G)$ called the abelian complex of G:

[1] This work was supported by NSF grant GP 9152.