

# THE ACTION OF A REAL SEMISIMPLE GROUP ON A COMPLEX FLAG MANIFOLD. I: ORBIT STRUCTURE AND HOLOMORPHIC ARC COMPONENTS<sup>1</sup>

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**1. Introduction.** This paper describes a topic that is of some interest in Lie groups and in differential and algebraic geometry. The topic shows good promise of being the “correct” context for explicit realization of those series of irreducible unitary representations of semisimple Lie groups that come into the Plancherel formula, so it probably is also of interest in harmonic analysis. We start with an example.

Let  $X$  be the Riemann sphere, viewed as  $\mathbf{C} \cup \{\infty\}$  via stereographic projection. Then the group  $G$  of all holomorphic automorphisms of  $X$  consists of the linear fractional transformations

$$\pm \begin{pmatrix} a & c \\ b & d \end{pmatrix} : z \rightarrow \frac{az + b}{cz + d}, \quad \det \begin{pmatrix} a & c \\ b & d \end{pmatrix} = 1;$$

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