ON INTEGRAL REPRESENTATIONS

BY ANDREAS DRESS

Communicated by Hyman Bass, April 1, 1969

Let G be a finite group and p a prime. G is called cyclic mod p if there exists a normal p-subgroup $N \trianglelefteq G$ such that G/N is cyclic.

Let R be a commutative ring with $1 \in R$. Write $\mathfrak{C}_R(G)$ for the set of subgroups $U \leqq G$, which are cyclic mod p for some appropriate prime p $(=p(U))$ with $pR \neq R$.

An RG-module M is a finitely generated R-module, on which G acts from the left by R-automorphisms. If $U \leqq G$ we write $M|_U$ for the RU-module, one gets by restricting the action of G on the R-module M to U.

If N is an RU-module, we write $N^{U \to G}$ for the induced RG-module $RG \otimes_{RU} N$.

Two RG-modules M, N are called weakly isomorphic, if there exists an RG-module L and a natural number k, such that $k \cdot M \oplus L \cong k \cdot N \oplus L$ ($k \cdot M$ short for $M \oplus \cdots \oplus M$, k times), we write then $M \simeq N$.

REMARK. If the Krull-Schmidt-Theorem holds for RG-modules, we have

$$M \simeq N \Leftrightarrow M \cong N.$$

THEOREM 1. *Let M, N be two RG-modules. If $M|_U \simeq N|_U$ for all $U \in \mathfrak{C}_R(G)$, then $M \simeq N$. Moreover there exist for any $U \in \mathfrak{C}_R(G)$ two R-free RG-modules $M(U)$, $N(U)$ with $M(U)|_V \cong N(U)|_V$ for all $V \leqq G$, which do not contain any conjugate of U, but $M(U)|_U \not\simeq N(U)|_U$.*

One can get an even more precise statement by using Grothendieck-rings: Let $X(G, R)$ be the Grothendieck-ring of RG-modules with respect to split-exact sequences, i.e. $X(G, R)$ is an as additive group isomorphic to the free abelian group, generated by the isomorphism classes of RG-modules modulo the subgroup generated by all expressions of the form $M - M_1 - M_2$ with $M \cong M_1 \oplus M_2$—and the multiplication in $X(G, R)$ is given by the tensor-product $\otimes_R$ of RG-modules. Write $X_Q(G, R)$ for $Q \otimes X(G, R)$. Obviously $M \simeq N$ if and only if M and N represent the same element in $X_Q(G, R)$.

$X(\cdot, R)$ and $X_Q(\cdot, R)$ are obviously contravariant functors from the category of groups into the category of commutative rings. Especially for $U \leqq G$ one has restriction homomorphisms $\mathrm{res}|_U$: $X(G, R) \to X(U, R)$, $X_Q(G, R) \to X_Q(U, R)$ and Theorem 1 reads now