A UNIVERSAL MODEL FOR DYNAMICAL SYSTEMS WITH QUASI-DISCRETE SPECTRUM

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Let K_d denote the circle group (unit interval with addition mod 1) with the discrete topology, let $\hat{K}_d$ be its dual (the Bohr compactification of the integers), and let $\hat{K}_d^\omega$ denote the direct product of countably many copies of $\hat{K}_d$. We define an affine transformation $\hat{\phi}$ of the group $\hat{K}_d^\omega$ by

$$\hat{\phi}(x_1, x_2, x_3, \cdots) = (x_1 + \hat{a}, x_1 + x_2, x_2 + x_3, \cdots),$$

where $\hat{a} \in \hat{K}_d$ is defined by

$$\hat{a}(t) = t \qquad (t \in K_d).$$

Implicit in the proof of the representation theorem of Halmos and von Neumann [5] for measure-preserving transformations with discrete spectrum is the fact that every such system is a factor (homomorphic image) of the translation $x \to x + \hat{a}$ on $\hat{K}_d$. Moreover, it follows easily from the fact that any subgroup Γ of K_d is the union of its finitely generated subgroups (as is any group) that any system with discrete spectrum is an inverse limit of direct products of ergodic translations on the circle group K (usual topology) and cyclic permutations of finite sets. A discussion of such inverse limits is given in [2]. In the present note we extend these two results to the ergodic dynamical systems Φ with quasi-discrete spectrum that were introduced by L. M. Abramov [1] in the measure-theoretic case and by F. Hahn and W. Parry [3] in the topological case.

In each case, by virtue of the respective representation theorems, we can assume without loss of generality that the system $\Phi = (G, \mathcal{B}, \mu, \phi)$ is such that G is a compact abelian group, $\mathcal{B}$ is the class of Baire subsets of G, μ is normalized Haar measure, and ϕ is an ergodic, continuous, affine transformation of G with quasi-discrete spectrum [1], [3], [4].

By an *affine* transformation of G we mean a map $\phi: G \to G$ of the form $\phi(x) = \tau(x) + a$, where τ is an automorphism of G and $a \in G$. Let us denote by σ the endomorphism of G defined by $\sigma(x) = \tau(x) - x$. Then ϕ is said to have *quasi-discrete spectrum* provided that

$$\bigcap_{n=1}^{\infty} \sigma^n G = (0).$$