ON SUMMABILITY FIELDS OF CONSERVATIVE OPERATORS

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Communicated by John W. Green, February 4, 1969

Let $B[c]$ denote the Banach algebra of all bounded linear operators on c, the set of convergent sequences. By a conservative operator we mean a member of $B[c]$. If $T\in B[c]$ and if there exists an infinite matrix $A=(a_{nk})$ such that $Tx=Ax$ for each $x\in c$, then T is called a conservative matrix. (By $Tx=Ax$ we mean $(Tx)_n=(Ax)_n\equiv\sum_k a_{nk}x_k$ for each $n\in I^+$, the set of positive integers.) Let Γ denote the subalgebra of $B[c]$ of all conservative matrices. If $T\in\Gamma$, its summability field, denoted by c_T, is taken to be the set $\{x\in s: Tx\in c\}$, where s denotes the set of all sequences. This raises the following question: How can one define the summability field c_T for an arbitrary T in $B[c]$? In other words, which sequences should one distinguish as being the set that a conservative operator sums?

One viewpoint is to consider how T acts on c_0, the maximal subspace of c consisting of those sequences which converge to 0. The restriction of T to c_0 is always representable by a matrix. In other words, if T' denotes the restriction of T to c_0, then there is an infinite matrix B so that $T'x=Bx$ for each $x\in c_0$. Surely, the summability field of T' is the set $c_B=\{x\in s: Bx\in c\}$. We now note that if T is a conservative matrix, say A, then A also represents the restriction of T to c_0, i.e. $A=B$. Thus, it seems reasonable to require that $c_T\supseteq c_B$ for any conservative operator T, where B is the matrix representing the restriction of T to c_0. Since the unit sequence $e=(1, 1, 1, \cdots)$ need not belong to c_B, even though Te always belongs to c, we cannot, in general, take $c_T=c_B$. However, since e is the only basis element of c that B might not sum, we propose that c_T be defined as

$$c_T = c_B \oplus e,$$

where $\oplus$ denotes the linear span of the sets c_B and e. The purpose then of this announcement is to report how the properties of c_T defined above for $T\in B[c]$ compare with the well-known properties of c_T for $T\in\Gamma$.

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[1] Research supported in part by NSF Grant GP-8199.

[2] Research supported in part by NSF Grant GP-8502.