THREE-MANIFOLDS WITH FUNDAMENTAL GROUP A FREE PRODUCT

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1. **Introduction.** The purpose of this paper is to announce some results concerning the structure of a compact 3-manifold M (possibly with boundary) where $\pi_1(M)$ is a free product. Related questions for M closed have been considered in [1], [2], [4], [6], [8].

We use the term *map* to mean continuous function. If M is a manifold, we use Bd M and Int M to stand for the boundary and interior of M, respectively. The disk D is said to be *properly embedded* in the 3-manifold M if

$$D \cap \text{Bd } M = \text{Bd } D.$$

The compact 3-manifold H_n is called a *handlebody of genus n* if H_n is the regular neighborhood of a finite connected graph having Euler characteristic $1-n$.

The combinatorial terminology is consistent with that of [9]. The terms in group theory may be found in [3]. Furthermore, all maps and spaces are assumed to be in the PL category.

2. **Bounded Kneser Conjecture.**

THEOREM 2.1. *Let M denote a compact 3-manifold with nonvoid boundary where $\pi_1(M) \approx A * B$, a free product. Then there is a compact 3-manifold M' with nonvoid boundary so that*

(i) *M' has the same homotopy type as M, and*

(ii) *there is a disk D' properly embedded in M' where $M'-D'$ consists of two components M_1 and M_2 with $\pi_1(M_1) \approx A$ and $\pi_1(M_2) \approx B$.*

OUTLINE OF PROOF. Let K_A and K_B denote CW-complexes with $\pi_1(K_G) \approx G$ and $\pi_n(K_G) = 0$, $n \geqq 2$, $G = A, B$. Let p denote a point not in $K_A \cup K_B$. Define $\overline{K}_A$ and $\overline{K}_B$ as the mapping cylinders of maps from p into K_A and K_B, respectively. Let K denote the CW-complex obtained by identifying the copy of p in $\overline{K}_A$ with the copy of p in $\overline{K}_B$. It follows that $\pi_1(K) \approx A * B$ and $\pi_n(K) = 0$, $n \geqq 2$ (see [1, p. 669]).

There is a simplicial map f of M into K (K may be chosen so that any finite collection of cells in K has a simplicial subdivision) so that f_* is an isomorphism of $\pi_1(M)$ onto $\pi_1(K)$.

LEMMA A. *Let M, K, f, p be as above. There is a map g: $M \rightarrow K$ so that*

(i) *g is homotopic to f relative to a base point of $\pi_1(M)$, and*