CERTAIN MAPPINGS OR DECOMPOSITIONS WHICH ARE TOPOLOGICALLY PROJECTIONS

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Introduction. A general question which is of interest is the following. Suppose that f is a mapping of a compact metric continuum X onto a metric space Y. *Under what conditions is there an embedding of X and Y in E^n (Euclidean n-space) or H^ω (Hilbert space) so that f is topologically equivalent to a projection onto Y defined by some collection of parallel hyperplanes?* Theorem 1 below provides an answer for a very special case of this general question. Although this theorem is actually a corollary of a more general theorem, we feel that its proof provides motivation and understanding for the main theorem.

THEOREM 1. *Suppose that U is the Universal* 1*-dimensional Menger Curve* [1] *and that f is a light open mapping of U onto I (the interval* [0, 1]) *such that $f^{-1}(x)$ is homeomorphic to a Cantor set for each x in I. Then there is a homeomorphism h of U into E^3 such that the mapping p defined by projecting U onto I through planes parallel to the yz-plane is topologically equivalent to f, that is, $ph=f$.*

We shall sketch a proof of this theorem. Our proof depends on an important theorem of J. H. Roberts [5] concerning contractibility in spaces of homeomorphisms, some very useful techniques of Dyer and Hamstrom [2], and a powerful selection theorem of E. A. Michael [4].

Statements of some results used in our proofs. Suppose that X is a compact metric space and dimension $X=n$ (an integer). For each positive integer k, let $H(X, I^k)$ be the space of all homeomorphisms of X into I^k (a k-cell) and let $C(X, I^k)$ be the space of all mappings of X into I^k. The metric, in each case, is the usual one: $\rho(f, g)=\max d(f(x), g(x))$ for x in X and d is the usual metric for I^k.

THEOREM (J. H. ROBERTS [5]). *Suppose that each of X and K is a compact metric space,* $\dim X=n$, $\dim K=r$, *and* $k\geqq 2n+2+r$. *Let α_0 and α_1 be mappings of K into $C(X, I^k)$. Then there exists a homotopy $f:K\times I\rightarrow C(X, I^k)$ such that*

(1) $f(\omega, 0)=\alpha_0(\omega), f(\omega, 1)=\alpha_1(\omega), \omega\in K$, *and*

(2) *for each t,* $0<t<1, f(\omega, t)\in H(X, I^k)$.

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