A THEORY OF COTYPES[1]

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1. **Introduction.** Suppose that M is a manifold, modeled on a Banach space X. For each subset S of M, we will define a cotype, which will be a nonnegative integer or ∞. The larger cotype S, the smaller S. The theory of cotypes will permit us to distinguish sizes of large subsets of M.

Cotype is a generalization of Baire category. Just as Baire's theory is based on sets whose closure contains no sphere, the theory of cotypes is based on sets whose closure contains no diffeomorph of a coball in X of specified codimension. We use the words diffeomorphism and diffeomorph in the following sense: ϕ is a C^q-diffeomorphism of M into a manifold N (modeled on a Banach space Y) iff ϕ is a bijective map, C^q in both directions, whose domain is open in M and whose range is open in N. If ϕ exists, then X and Y must be isomorphic. If a special term is desired, ϕ may be called a full diffeomorphism.

As an application, we cite a theorem on the cotype of the image of the critical set of a Fredholm map of negative index. The theorem is suggested by and is close to one of Smale's.

The definition of cotype will be relative to a differentiability class C^q. The reader may wish to take $q=1$ throughout. It is quite possible that cotype is independent of q, for $q\geqq 1$. That is, if M is a C^q-manifold, $q\geqq 1$, modeled on X, and if $S\subset M$, then cotype S may be unchanged if we increase q by changing the atlas of M. This is likely to be the case, if X is finite dimensional [4]. Our theory is directed, however, towards infinite dimensional manifolds. Cotype for $q=0$ may be different from cotype for $q=1$.

2. **Cotype.** Let M be a C^q-manifold, with countable basis, modeled on a Banach space X of more than one point, with or without boundary.

By a set $\gamma_r\subset M$, we shall always mean a diffeomorph of an r-coball, $r=0, 1, \cdots$. That is,

$$\gamma_r = \phi\Gamma_r,$$

where ϕ is a C^q-diffeomorphism of X into M, and Γ_r is an r-coball,

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